# CS 395/495-26: Spring 2002

# IBMR: Week 10B

## Epipolar Geometry and Conclusions
Chapter 8

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## Reminders

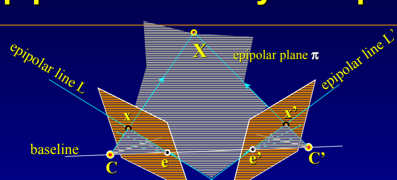
CTEC Online – please add your comments…

- Homework 1 return
- Proj3 Due Thurs May 23

  HW2 posted on website.
- HW2 Due Thurs May 30

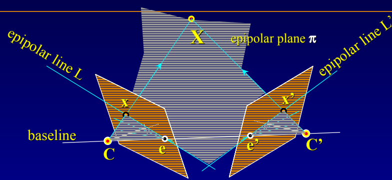
  Proj4 posted on website.
- Proj4 Due Tues June 11

## Epipolar Geometry: Chapter 8




**Summary:**
- Connect cameras C, C' with a baseline, which hits image planes at epipoles e, e'.
- Chose any world pt **X, then →→** everything is coplanar!
  epipolar plane π includes image points x, x', and these connect to epipoles e,e' by epipolar lines L, L'

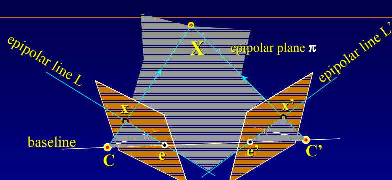
## Epipolar Geometry




**Useful properties:**

- Epipole e' = 2nd camera's (image of 1st camera C)
- All epipolar lines L' pass through epipole e'
- Epipolar Line L' is (image of **C→X** ray...)
- Epipolar Line L' links (image of **C**) to (image of **X**)
- Every image point x maps to an epipolar line L'

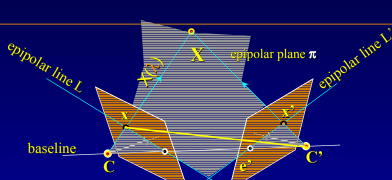
## Fundamental Matrix: Fx = L'




One Matrix Summarizes ALL of epipolar geometry

- Maps image point x to epipolar line L': F x = L'
- How? use full 3x4 camera matrices P, P' and …

## Fundamental Matrix: Fx = L'

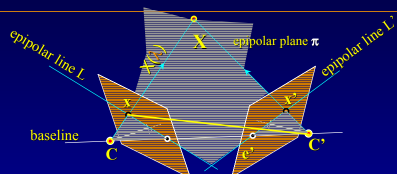



- Recall Pseudo-Inverse: $P^+ = P^T (P P^T)^{-1}$ and $P P^+ = I$ (pg148)
- Write world-space ray C→X as: $X(\lambda) = P^+x + \lambda C$
- Other camera's image of the ray is its epipolar line L':

  $L'(\lambda) = P'X(\lambda) = P'P^+x + \lambda P'C$ .
- But P'C = e'; it is the epipole of the other camera, so

  $L'(\lambda) = P'P^+x + \lambda e'$

## Fundamental Matrix: Fx = L'




- But $P'C = e'$; it is the epipole of the other camera, so

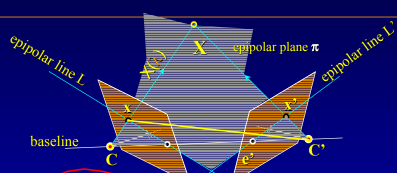
  $L'(\lambda) = P'P^{+}x + \lambda e'$

- Rewrite epipolar line L' using points $P'P^{+}x$ and e'
- (Recall: find line between two $P^2$ points with cross product: $L = x_1 \times x_2$)
- To get

  $L' = e' \times P' P^{+} x$ or $L' = F x$

THUS $F = [e']_{\times} P' P^{+}$ ← ?!?!What?!?! cross product with a matrix?!?!

## Fundamental Matrix: Fx = L'




$F = [e']_{\times} P' P^{+}$ But what's this? NEW TRICK:

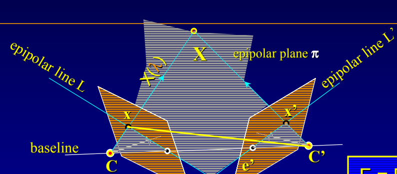
- Cross Product written as matrix multiply (pg. 554)

$$a \times b = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \times \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = [a]_{\times} \cdot b$$

- Note: $a \times b = -b \times a = [a]_{\times} \cdot b = (a^T \cdot [b]_{\times})^T$

'skew symmetric' matrix

## Fundamental Matrix: Fx = L'




$F = [e'] \times P' P^{+}$

- Matrix F is unique for a point pair (up to scaling)
- Cool! works even for different cameras!
- F tied DIRECTLY to corresp. point pairs (x, x'):
  - F finds epipolar line L' from point x : $Fx = L'$
  - (Recall that if (any) point x' is on line a L', then $x'^{T} L' = 0$)
  - Substitute Fx for L': $x'^{T} F x = 0$

## Fundamental Matrix Properties

(pg. 226)

- F is 3x3 matrix, maps $P^2 \rightarrow P^2$, rank 2, 7-DOF
- If world space pt X → image space pts. x and x' then
  $x'^T F x = 0$
- Every image pt has epipolar line in the other image:
  $Fx = L'$ $F^T x' = L$
- Baseline pierces image planes at epipoles e, e' :
  $Fe = 0$ $F^T e' = 0$
- Given camera matrices P, P', find F matrix by:
  $F = [e']_\times P' P^+$ (recall: e' is image of C: e' = P'C)
- F is unaffected by any world-space proj. transform
  (PH, P'H) has same F matrix as (P, P') for any full-rank H
  (in other words, choose any world-space axes you like)

## Fundamental Matrix Uses

Special case: camera translate only (no rotations)

- Camera matrices are $P = K[\,\mathbf{I} \mid \mathbf{0}\,]$, $P' = K[\mathbf{I} \mid \mathbf{t}\,]$
  - where K is internal calib., **t** is 3D translation vector $\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$
- F matrix simplifies to $F = [e']_\times$
- Epipolar lines are all parallel to direction **t**
- x,x' displacement depends only on **t** & 3D depth z:
  $x' = x + (K\mathbf{t})(1/z)$

## Fundamental Matrix Uses

General movement?

- Recall: rotations don't change image content
  (camera rotate→projective image warp H)
- ANY cameras, ANY movements can then be warped to remove rotations, THUS
- Can ALWAYS get parallel epipolar lines!
  - Easier to find correspondences
  - Easier to find depth values z
  - 'Parallel Epipolar Lines'=='Rectified Image Pair'

## Fundamental Matrix Properties

(pg. 226)

Why bother with F?

- Can find it from image pt. correspondences only
- Works even for mismatched cameras
  (example: 100-year time-lapse of Eiffel tower)
- Choose your own world-space coordinate system.
- SVD lets us recover P, P' camera matrices from F
  - (4-way ambiguity; what is front/back of C and C'?) pg 240
  - BUT WE DON'T NEED TO!
- Complete 2-camera mapping from world←→image
  - 2 images + corresponding point pairs $(x_i, x'_i) \rightarrow F$
  - Let camera coords == 3D world coords, then $(x_i, x'_i) \rightarrow X_i$

## Conclusions

- $P^2$, $P^3$ matrix forms give elegant, principled notation for ALL image geometry
  - Cameras, lights, points, lines, planes, conics, quadrics, twisted cubics, …
  - Matrix form makes everything reversible: 3D from (2D)*!
  - Shape recovery from point correspondence: DONE.
- Light/Surface interactions are linear too:
  - (illumination)*(reflectance) = light from surface
  - Challenge: recover shape AND reflectance from images
  - Difficulty: reflectance changes with angle; so does illum.
  - Challenge: automatic point correspondence despite ↺
  - Challenge: motion in scene, streaming images, …
  - Challenge: full 8-dim. light field recovery with shape…

## Conclusions

IBMR Course 1st Attempt:

- Too much CV, not enough CG & apps
- Covered strong, best, but toughest part of IBMR
  - Now you can understand, reproduce most current IBMR papers
  - Example: Marc Pollifey's SIGG`99 Course "3D photography"
- Skipped ugly, tedious, unreliable parts of CV:
  - Given an image, measure the best 2D points, lines, conics…
  - Correspondence finding; resolution, resampling & bandwidth
- This course was too hard! I'll fix that …
  Thank you for patience & hard work;
  you helped develop a substantial new course.

# END