CS 395/495-26: Spring 2002

IBMR: Week 10A

The Camera Matrix and World Geometry

Chapter 7 (finish)

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Reminders

CTEC Online – please add your comments…

- Homework 1 return
- Proj3 Due Thurs May 23

  HW2 posted on website.
- HW2 Due Thurs May 30

  Proj4 posted on website.
- Proj4 Due Tues June 11

Cameras as Protractors

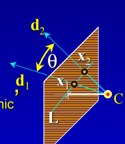
- Image Direction: $d = [x_c, y_c, z_c, 0]^T$
- Image Direction from a point **x**: $d = K^{-1}x$
- Angle θ between C and 2 image points x_1, x_2:

$$\cos\theta = \frac{x_1^T (K^{-T}K^{-1})\, x_2}{\sqrt{(x_1^T (K^{-T}K^{-1})\, x_1)(x_2^T (K^{-T}K^{-1})\, x_2)}}$$

(pg 199)

- Simplify with absolute conic Ω_∞:

$P\,\Omega_\infty = (K^{-T}K^{-1}) = \omega$ = 'Image of Absolute Conic'



Cameras as Protractors

$P\,\Omega_\infty = (K^{-T}K^{-1})$. OK. Now what was Ω_∞ again?

Recall P^3 Conic Weirdness: (pg. 63-67)

- Plane at infinity π_∞ holds all 'horizon points' d ('universe wrapper')
- Absolute Conic Ω_∞ imaginary points in outermost circle of π_∞
  - Satisfies BOTH $x_1^2 + x_2^2 + x_3^2 = 0$ AND $x_4^2 = 0$
  - Can rewrite equations to look like a quadric (but isn't— no x_4)

$$\begin{bmatrix} x_1 & x_2 & x_3 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ 0 \end{bmatrix} = \mathbf{d}^T \cdot \Omega_\infty \cdot \mathbf{d}$$

  - AHA! 'points' on it are (complex conjugate) directions **d** !
- Finds right angles-- if $d_1 \perp d_2$, then: $\mathbf{d}_1^T \cdot \Omega_\infty \cdot \mathbf{d}_2 = 0$

Cameras as Protractors

$P\,\Omega_\infty = (K^{-T}K^{-1})$. OK. Now what was Ω_∞ again?

- Dual of Absolute Conic Ω_∞ is Dual Quadric Q^*_∞ (?!?!)
- More compact notation: for imaginary planes π

$$\begin{bmatrix} \pi_1 & \pi_2 & \pi_3 & \pi_4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix} = \pi^T \cdot Q^*_\infty \cdot \pi$$

- Same matrix, but different use:
  - --find a plane π for every possible direction d
  - --π is $\perp$ to π_∞, and tangent to the quadric Q^*_∞
- Ω_∞ is circle in π_∞ where tangent planes π are $\perp$ to π_∞
- Finds right angles-- if $\pi_1 \perp \pi_2$, then: $\pi_1^T \cdot Q^*_\infty \cdot \pi_2 = 0$

Cameras as Protractors

$P\,\Omega_\infty = (K^{-T}K^{-1}) = \omega$ = 'Image of Absolute Conic'

- Just Ω_∞ as has a dual $\mathbf{Q}^*_\infty$, ω has dual ω^* :

$$\omega^* = \omega^{-1} = K\,K^T$$

- The dual conic ω^* is the image of $\mathbf{Q}^*_\infty$, so

$$\omega^* = P\,\mathbf{Q}^*_\infty = P \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \text{1st 3 columns of P?}$$

Cameras as Protractors

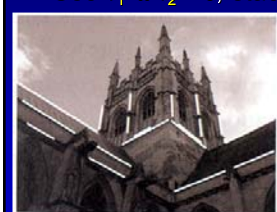
$P\,\Omega_\infty = (K^{-T}K^{-1}) = \omega$ = 'Image of Absolute Conic'

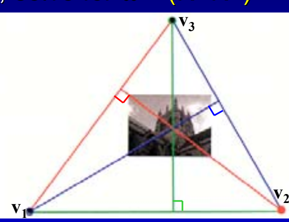
- Just Ω_∞ as has a dual $\mathbf{Q}^*_\infty$, ω has dual ω^* :
  $\omega^* = \omega^{-1} = K\,K^T$
- The dual conic ω^* is the image of $\mathbf{Q}^*_\infty$, so
  $\omega^* = P\,\mathbf{Q}^*_\infty = P \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ = 1st 3 columns of P?
- Vanishing points v_1, v_2 of 2 ⊥ world-space lines:
  $v_1^T \omega\, v_2 = 0$
- Vanishing lines L_1, L_2 of 2 ⊥ world-space planes:
  $L_1^T \omega^* L_2 = 0$

Cameras as Protractors

Clever vanishing point trick:

- Perpendicular lines in image?
- Find their vanishing pts. by construction:
- Use $v_1^T \omega\, v_2 = 0$, stack, solve for $\omega = (K^{-T}K^{-1})$

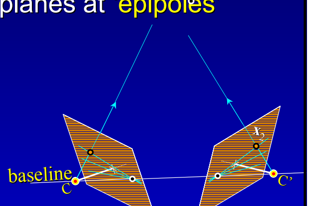




Epipolar Geometry: Chapter 8

Basic idea:

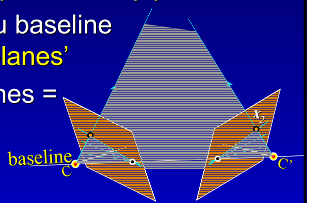
- 2 cameras centered at C, C' in world space
- Draw 'baseline' through camera centers
- Baseline hits image planes at 'epipoles'



Epipolar Geometry: Chapter 8

Basic idea:

- 2 cameras centered at C, C' in world space
- Draw 'baseline' through camera centers
- Baseline hits image planes at 'epipoles'
- Family of planes thru baseline all are 'epipolar planes'
- Image of planes = lines = 'epipolar lines'
- Lines intersect at epipolar point.



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