CS 395/495-26: Spring 2002

IBMR: Week 8B

$R^3 \rightarrow R^2$ and $P^3 \rightarrow P^2$
The Camera Matrix

Jack Tumblin
jet@cs.northwestern.edu

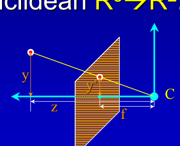
Reminders

- **HW1 delayed: due May 21**
- Proj3 Due today, Thurs May 23
  HW2 posted on website.
- HW2 due Thurs May 30
  Proj4 posted on website.
  HW 3 Assign Tues May 28
- Proj4 Due Tues June 11
- HW3 Due Tues June 11

Cameras Revisited

- Goal: Formalize projective 3D→2D mapping
- Homogeneous coords handles infinities well
  - Projective cameras (convergent 'eye' rays)
  - Affine cameras (parallel 'eye' rays)
  - Composed, controlled as matrix product
- But First: Cameras as Euclidean $R^3 \rightarrow R^2$:

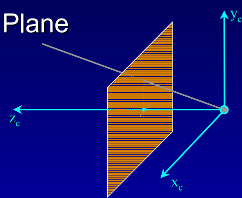
$x' = f\, x / z$
$y' = f\, y / z$



Cameras Revisited

Plenty of Terminology:

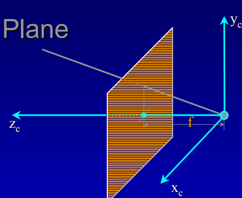
- Image Plane or Focal Plane
- Focal Distance f
- Camera Center C
- Principal Point
- Principal Axis
- Principal Plane (?!?! DOESN'T touch principle point!?!?)
- Camera Coords (x_c,y_c,z_c)
- Image Coords (x',y')



Cameras Revisited

Plenty of Terminology:

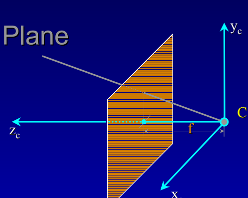
- Image Plane or Focal Plane
- Focal Distance f
- Camera Center C
- Principal Point
- Principal Axis
- Principal Plane (?!?! DOESN'T touch principle point!?!?)
- Camera Coords (x_c,y_c,z_c)
- Image Coords (x',y')



Cameras Revisited

Plenty of Terminology:

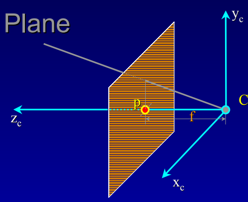
- Image Plane or Focal Plane
- Focal Distance f
- Camera Center C
- Principal Point
- Principal Axis
- Principal Plane (?!?! DOESN'T touch principle point!?!?)
- Camera Coords (x_c,y_c,z_c)
- Image Coords (x',y')



Cameras Revisited

Plenty of Terminology:

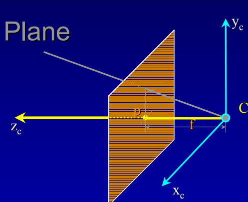
- Image Plane or Focal Plane
- Focal Distance f
- Camera Center C
- Principal Point p
- Principal Axis
- Principal Plane (?!?! DOESN'T touch principle point!?!?)
- Camera Coords (x_c, y_c, z_c)
- Image Coords (x', y')



Cameras Revisited

Plenty of Terminology:

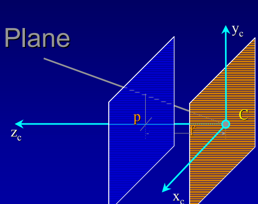
- Image Plane or Focal Plane
- Focal Distance f
- Camera Center C
- Principal Point p
- Principal Axis
- Principal Plane (?!?! DOESN'T touch principle point!?!?)
- Camera Coords (x_c, y_c, z_c)
- Image Coords (x', y')



Cameras Revisited

Plenty of Terminology:

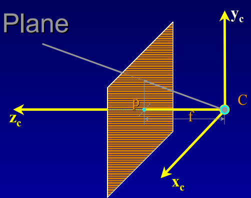
- Image Plane or Focal Plane
- Focal Distance f
- Camera Center C
- Principal Point p
- Principal Axis
- Principal Plane (?!?! DOESN'T touch principle point P !?!?)
- Camera Coords (x_c, y_c, z_c)
- Image Coords (x', y')



Cameras Revisited

Plenty of Terminology:

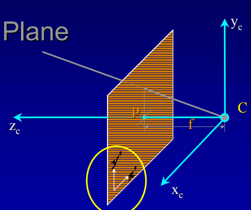
- Image Plane or Focal Plane
- Focal Distance f
- Camera Center C
- Principal Point p
- Principal Axis
- Principal Plane (?!?! DOESN'T touch principle point!?!?)
- Camera Coords (x_c, y_c, z_c)
- Image Coords (x',y')



Cameras Revisited

Plenty of Terminology:

- Image Plane or Focal Plane
- Focal Distance f
- Camera Center C
- Principal Point p
- Principal Axis
- Principal Plane (?!?! DOESN'T touch principle point!?!?)
- Camera Coords (x_c, y_c, z_c)
- Image Coords (x',y')

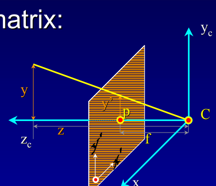


Homogeneous Coords: $R^3 \rightarrow P^2$

- Basic Camera as a 3x4 matrix:

$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

$$P_0 \, X = x$$



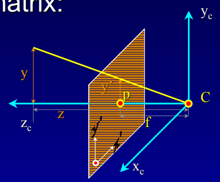
- Tricky! **X** is in augmented R^3, not P^3 (yet)
  x is in P^2 space
- As shown: origin of (x',y') is principal point **p**, but pixel counting starts at corners…

Homogeneous Coords: $R^3 \rightarrow P^2$

- Basic Camera as a 3x4 matrix:

$$\begin{bmatrix} f & 0 & p_x & 0 \\ 0 & f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

$$P_0 \; X \; = x$$



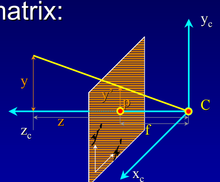
- Translate image coords (x',y'):
  Shifts principal point p from (0,0) to (p_x,p_y)
  - is NOT obvious: scales z_c
  - does NOT use 'homogeneous' 1 term in **X**

Homogeneous Coords: $R^3 \rightarrow P^2$

- Basic Camera as a 3x4 matrix:

$$\begin{bmatrix} \alpha_x f & s & p_x & 0 \\ 0 & \alpha_y f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

K (3x3 submatrix) $\quad P_0 \; X \; = x$



Less common adjustments:

- Non-square pixels? change scaling (α_x, α_y)
- Parallelogram pixels? set nonzero skew **s**

K matrix: "(internal) camera calib. matrix"

Matrix Form: Euclidean $R^3 \rightarrow R^2$

- K matrix: "internal camera calib. matrix"
- R·T matrix: "external camera calib. matrix"
  - T matrix: Translate world to cam. origin
  - R matrix: 3D rotate world to fit cam. axes

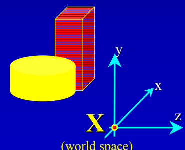
Input: **X** 3D World Space → Output: **x** 2D Camera Image

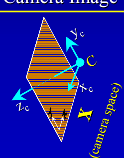
Combine: write

$(P_0 \cdot R \cdot T) \cdot X = x$

as

$P \cdot X = x$





Uses for Camera Matrix P

$\mathbf{P \cdot X = x}$, or $\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \mathbf{P} & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ t_w \end{bmatrix} = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$ $\quad \mathbf{P} = \left[\begin{array}{c|c} \mathbf{M} & p_4 \end{array} \right]$

- P's Null Space: camera center $\tilde{C}$ in world space!

  $P\,\tilde{C} = 0$ (solve for C)

  - Finite Camera: $\tilde{C} = \begin{bmatrix} -\mathbf{M}^{-1} \cdot \mathbf{p}_4 \\ \hline 1 \end{bmatrix}$ Affine Camera: $\tilde{C} = \begin{bmatrix} d \\ \hline 0 \end{bmatrix}$ (where $Md=0$)

- Surprise! (minor) works projectively too: $P^3 \rightarrow P^2$

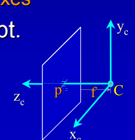
  (see book pg. 152-3 for restrictions—almost none!)

Uses for Camera Matrix P

$\mathbf{P \cdot X = x}$, or $\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \mathbf{P} & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ t_w \end{bmatrix} = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$ $\quad \mathbf{P} = \left[\begin{array}{c|c|c|c} \mathbf{P}^1 & \mathbf{P}^2 & \mathbf{P}^3 & \mathbf{P}^4 \end{array} \right]$

Columns of P matrix: Points in image-space:

- P^1, P^2, P^3 == image of x,y,z axis vanishing points
  - Proof: let $D = [1\ 0\ 0\ 0]^T$ = point on x axis, at inifinity
  - PD = 1st column of P. Repeat for y and z axes
- P^4 == image of the world-space origin pt.
  - Proof: let $\mathbf{D}$ = [0 0 0 1]T = world origin
  - $\mathbf{PD}$ = 4th column of $\mathbf{P}$ = image of origin pt.

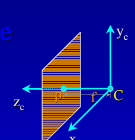


Uses for Camera Matrix P

$\mathbf{P \cdot X = x}$, or $\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \mathbf{P} & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ t_w \end{bmatrix} = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$ $\quad \mathbf{P} = \begin{bmatrix} \mathbf{p}^{1T} \\ \hline \mathbf{p}^{2T} \\ \hline \mathbf{p}^{3T} \end{bmatrix}$

Rows of P matrix: planes in world space

- row 1 = $\mathbf{p}^{1T}$ = image x-axis plane
- row 2 = $\mathbf{p}^{2T}$ = image y-axis plane
- row 1 = $\mathbf{p}^{3T}$ = camera's principal plane

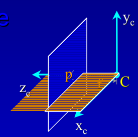


Uses for Camera Matrix P

$\mathbf{P} \cdot \mathbf{X} = \mathbf{x}$, or $\begin{bmatrix} & & & \\ & \mathbf{P} & & \\ & & & \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ t_w \end{bmatrix} = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$ $\quad \mathbf{P} = \begin{bmatrix} \mathbf{p}^{1T} \\ \mathbf{p}^{2T} \\ \mathbf{p}^{3T} \end{bmatrix}$

Rows of P matrix: planes in world space

- row 1 = $\mathbf{p}^1$ = image x-axis plane
- row 2 = $\mathbf{p}^2$ = image y-axis plane
- row 1 = $\mathbf{p}^3$ = camera's principal plane

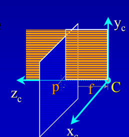


Uses for Camera Matrix P

$\mathbf{P} \cdot \mathbf{X} = \mathbf{x}$, or $\begin{bmatrix} & & & \\ & \mathbf{P} & & \\ & & & \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ t_w \end{bmatrix} = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$ $\quad \mathbf{P} = \begin{bmatrix} \mathbf{p}^{1T} \\ \mathbf{p}^{2T} \\ \mathbf{p}^{3T} \end{bmatrix}$

Rows of P matrix: planes in world space

- row 1 = $\mathbf{p}^1$ = image x-axis plane
- row 2 = $\mathbf{p}^2$ = image y-axis plane
- row 1 = $\mathbf{p}^3$ = camera's principal plane

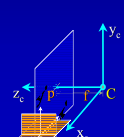


Uses for Camera Matrix P

$\mathbf{P} \cdot \mathbf{X} = \mathbf{x}$, or $\begin{bmatrix} & & & \\ & \mathbf{P} & & \\ & & & \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ t_w \end{bmatrix} = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$ $\quad \mathbf{P} = \begin{bmatrix} \mathbf{p}^{1T} \\ \mathbf{p}^{2T} \\ \mathbf{p}^{3T} \end{bmatrix}$

Rows of P matrix: planes in world space

- row 1 = $\mathbf{p}^1$ = image x-axis plane
- row 2 = $\mathbf{p}^2$ = image y-axis plane
- Careful! Shifting image origin shifts the x,y axis planes!

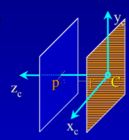


Uses for Camera Matrix P

$\mathbf{P} \cdot \mathbf{X} = \mathbf{x}$, or $\begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \mathbf{P} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ t_w \end{bmatrix} = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$ $\quad \mathbf{P} = \begin{bmatrix} \mathbf{p}^{1T} \\ \mathbf{p}^{2T} \\ \mathbf{p}^{3T} \end{bmatrix}$

Rows of P matrix: planes in world space

- row 1 = $\mathbf{p}^1$ = image x-axis plane
- row 2 = $\mathbf{p}^2$ = image y-axis plane
- row 3 = $\mathbf{p}^3$ = camera's principal plane



END