CS 395/495-26: Spring 2002

IBMR: Week 5 B

Chapter 3: Estimation & Accuracy

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Homework 1

- Paper-and-pencil exercises
- Shouldn't be very hard—mostly practice and reminders of important properties
- Not comprehensive (skips good stuff)
- Due in 2 weeks: May 16
- (Turn it in! only got 2 Proj 1's…)

Overview

Perfect Math .vs. Imperfect measurements

- **Many** ways to recover **H**:
  Points, lines, planes, ||, ⊥, conics, quadrics, cross-ratios, vanishing points, twisted cubics…
- **Vectorize** ('flatten, stack, null space') to solve most / all
- **Robustness**, Accuracy .vs. # of measurements (data-rich images: **quantity** easier than **quality**
  more is easier than **better**)

?What links measurement errors←→**H** errors?
?How can more measurements reduce error?

Recall: Project 2 Hints

4 point correspondence:

- Book shows: $x'(h_{31}x + h_{32}y + h_{33}) = (h_{11}x + h_{12}y + h_{13})$
  $y'(h_{31}x + h_{32}y + h_{33}) = (h_{11}x + h_{12}y + h_{13})$
- Rearrange: known vector (dot) unknown vector

$$\begin{bmatrix} x & y & 1 & 0 & 0 & 0 & -x'x & -x'y & -x' \\ 0 & 0 & 0 & x & y & 1 & -y'x & -y'y & -y' \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = 0$$

- stack, solve for null space…

But this assumes x_1=x, x_2=y, x_3=1,
Book has a better way (DLT)…

Direct Linear Tranform (DLT)

- Project 2 method: Solve Hx – x' = 0
  - Requires constant scale (x_3=1 in measurement)

$$\begin{bmatrix} 0 & 0 & 0 & x & y & 1 & -y'x & -y'y & -y' \\ x & y & 1 & 0 & 0 & 0 & -x'x & -x'y & -x' \end{bmatrix}$$

- **compare:**

$$\begin{bmatrix} 0 & 0 & 0 & -w'x & -w'y & -w'w & y'x & y'y & y'w \\ w'x & w'y & w'w & 0 & 0 & 0 & -x'x & -x'y & -x'w \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = 0$$

- DLT method: Solve $Hx \times x' = 0$
  - Accepts any scale (x_3=w≠1 OK, xx_3)

Direct Linear Tranform (DLT)

- Project 2 method: Solve Hx – x' = 0
  - Requires constant scale (x_3=1 always)

$$\begin{bmatrix} 0 & 0 & 0 & x & y & 1 & -y'x & -y'y & -y' \\ x & y & 1 & 0 & 0 & 0 & -x'x & -x'y & -x' \end{bmatrix}$$

- **compare:**

$$\begin{bmatrix} 0 & 0 & 0 & -w'x & -w'y & -w'w & y'x & y'y & y'w \\ w'x & w'y & w'w & 0 & 0 & 0 & -x'x & -x'y & -x'w \end{bmatrix}$$

$$\begin{bmatrix} 0^T & -w'_i \mathbf{x}_i^T & y'_i \mathbf{x}_i^T \\ w'_i \mathbf{x}_i^T & 0^T & -x'_i \mathbf{x}_i^T \end{bmatrix} \begin{bmatrix} \mathbf{h}^1 \\ \mathbf{h}^2 \\ \mathbf{h}^3 \end{bmatrix} = 0$$

- DLT method: Solve $Hx \times x' = 0$
  - Accepts **any** scale (x_3=w≠1, x_3'=w'≠1)

Direct Linear Tranform (DLT)

- DLT method: Solve H $\times$ x = 0
  - Accepts any scale, any point (x_3=w≠1 is OK)
  - 'Pure', Compatible -- $\mathbf{P}^2$ terms only
  - Much better suited to error measurements.
- Subtlety:
  - Won't divide-by-zero if w=0 or h_{33}=0 (it happens!)
  - has a 3rd row; it is not degenerate if w'=0
  - OK to use it… (Solve 8x12)

$$\begin{bmatrix} 0^T & -w'_i \mathbf{x}_i^T & y'_i \mathbf{x}_i^T \\ w'_i \mathbf{x}_i^T & 0^T & -x'_i \mathbf{x}_i^T \\ -y'_i \mathbf{x}_i^T & -x'_i \mathbf{x}_i^T & 0^T \end{bmatrix} \begin{bmatrix} \mathbf{h}^1 \\ \mathbf{h}^2 \\ \mathbf{h}^3 \end{bmatrix} = 0$$

Deceptive 'Robustness'

- Suppose we have 4 pt-correspondences:
  - Use DLT to write 8x9 (or 12x9) matrix **A**: Ah=0
  - Solve for h null space. ALWAYS gives **H** matrix
- But what if points are bad / fictional?
  - 3 collinear input pts, crooked out: **IMPOSSIBLE!**
  - Yet we get an **H** solution! Why?
- **A** matrix rank is 7 or 6→ rank 1 **H** result(s)
  - 'Null Space' of **A** may contain >1 **H** solution!
  - 'Degenerate' **H** solution of form $a\mathbf{H}_1 + b\mathbf{H}_2$…
- Answer: SVD ranks **A**; reject bad point sets.

Actual 'Robustness'

- Vectorizing ('Flatten, Stack, Null Space' method) works for almost ANY input!
  (Points, lines, planes, ||, ⊥, conics, quadrics, cross-ratios, vanishing points, twisted cubics…
- Use DLT formulation: (Hx $\times$ x') = 0

  measured input transformed $\times$ measured output = 0
- Rearrange as dot product: (known)·(unknown) = 0
- Be careful to have ENOUGH constraints
  tricky when you mix types: points, lines,… (pg 75)

Adding More Measurements

How can we use >4 point correspondences?

- Easy:
  - Add more rows to our 8x9 matrix A: **A·h = 0**
  - Use SVD to find Null space (Always gives an answer!)
  - **Result:** 'Least squares' solution
    - minimizes $\| \mathbf{A}\,\mathbf{h} \|^2 = \Sigma_i\, \varepsilon_i$
      where ε_i is error for i-th pt. correspondence:
    - $\varepsilon_i = \| Hx_i \times x_i' \|^2 = \|$ (2 rows of A)*h $\|^2$ = 'algebraic distance'
  - **'Algebraic Distance' ?** No geometric meaning!

Adding More Measurements

- **2D'Algebraic Distance' ?** No geometric meaning!
- **2D 'Geometric Distance' d(a,b)²** is Better:
  measurable length in input or output space
  if $a = (a_1\ a_2\ a_3)$ and $b = (b_1\ b_2\ b_3)$, then define

$$\mathbf{d}(a,b)^2 = \left(\frac{a_1}{a_3} - \frac{b_1}{b_3}\right)^2 + \left(\frac{a_2}{a_3} - \frac{b_2}{b_3}\right)^2$$

Turns out that:

$$\mathbf{d}(a,b)^2 = \frac{d_{algebraic}(a,b)}{a_3 \cdot b_3}$$

(Not very surprising)

Adding More Measurements

Overall Strategy:

- Overconstrain the answer **H**
  - Collect extra measurements (>4 point pairs, etc. …)
  - expect errors; call them 'estimates' $\hat{x}$
- Compute a 1st solution (probably by SVD)
- Compute error $d(H\hat{x}, \hat{x}')^2$, and use this to…
- 'Tweak' answer H **and** estimates $\hat{x}$
- Compute new answer
- Stop when error < useful threshold

Using Estimates

- Simplest: 'one image' transfer method:
  - Assume inputs are a perfect test pattern: only output pts are in error
  - Adjust output estimates $\hat{x}'$ until $d(Hx, \hat{x}')^2 \to 0$ (note we re-compute H as x' changes)
- Better: 'Symmetric' transfer method:
  - Assume BOTH inputs and outputs have error.
  - Adjust BOTH input and output ests $\hat{x}$ $\hat{x}'$ (note we re-compute H as x, x' change)
  - Stop when $d(H\hat{x},\hat{x}')^2 + d(H^{-1}\hat{x}', \hat{x})^2 \to 0$

END