2 (p276).
\(a\) There are \(c\) children choosing from among \(d\) kinds of candy. The probability that a given variety of candy is chosen by no child is: 
\[
\left(1 - \frac{1}{d}\right)^c
\]

\(b\) The expected number of kinds of candy chosen by no child is: 
\[
d \left(1 - \frac{1}{d}\right)^c
\]

\(c\) If \(c = d\), then the expected number of kinds of candy chosen by no child changes. From (b), we know that if \(c = d\), the expected number is: 
\[
d \left(1 - \frac{1}{d}\right)^d
\] 
As \(n\) grows, we know that 
\[
\text{Limit} \left[(1 - \frac{1}{d})_d \rightarrow \infty\right] = e
\] 
so the expected number is: 
\[
ed
\]

8 (p276)
\(a\) The probability that all \(n\) items hash to different locations (no collisions) is: 
\[
\frac{(k*(k-1)*...*(k-n+1))}{k^n}
\]

\(b\) The probability that the \(i\)th item is the first collision is: 
\[
1 - \frac{(k*(k-1)*...*(k-i+1))}{k^i}
\]

\(c\) Since there are two outcomes, collision or no-collision, the probability that the \(i\)th item collides is: 
\[
\frac{(i-1)}{k}
\]

The expected number of elements hashed before the first collision is thus: 
\[
\frac{k}{(i-1)}
\] 
where \(i\) is the number of already filled in elements without collision.

d) 

4 (p292)
\[
T(n) = \frac{1}{4}(S1 + T(n)) + \frac{1}{4}(S2) + \frac{1}{4}(S3) + \frac{1}{4}(S4)
\]
\[
3/4(T(n)) = $2.50
\]
\[
T(n) = $3.33
\]

Maximum amount of money he’d be willing to pay is $3.33 as that is the expected payout.
8 (p293) Prove Theorem 5.23:
\[ \sum_i E(X \mid F_i) P(F_i) =. \]
\[ \sum_i \sum_k x_k P(X = x_k \mid F_i) P(F_i) =. \]
\[ \sum_i \sum_k x_k P(X = x_k, F_i, occurs) =. \]
\[ \sum_k \sum_i x_k P(X = x_k, F_i, occurs) =. \]
\[ \sum_k x_k P(X = x_k) =. \]

\[ E(X) \]

2 (p305)
The expected value of \( X_i \) is \( \frac{3}{5} \). The variance of \( X_i \) is \( \frac{3}{5} \left( 1 - \frac{3}{5} \right)^2 + \frac{2}{5} \left( 0 - \frac{3}{5} \right)^2 = \frac{6}{25} \). The sum of the variances of \( X_1 \ldots X_5 \) is \( \frac{30}{25} = \frac{6}{5} \), which is identical to the variance \( X \) of Problem 1.

4 (p305)
The expected number of right answers is 60. The variance is \( 100 \left( \frac{6}{25} \right) = 24 \), and the standard deviation is \( \sqrt{24} \).

Extra Credit

16 (p276)

10 (p293)

6 (p305)
The variance for one question is \( \frac{4}{5} \left( 1 - \frac{4}{5} \right)^2 + \frac{1}{5} \left( 0 - \frac{4}{5} \right)^2 = \frac{4}{25} \), so for 25 questions it is 4. If 100 questions, 16, and if 400 questions, 64. To represent the spread better, you can take the standard deviation (which is taking the square root of the variance).