Rhythm Analysis in Music

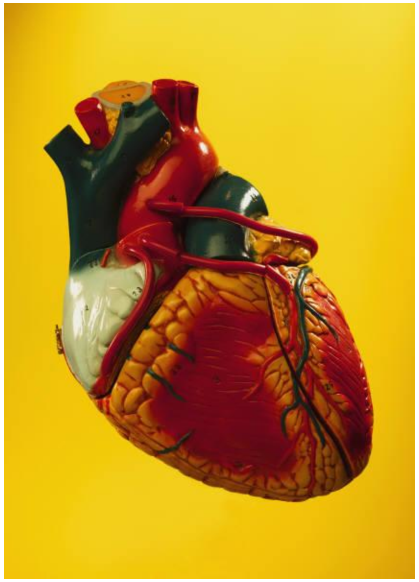
EECS 352: Machine Perception of Music & Audio

Some Definitions

- Rhythm
  - "movement marked by the regulated succession of strong and weak elements, or of opposite or different conditions." [OED]

Some Definitions

- Beat
  - Basic unit of time in music

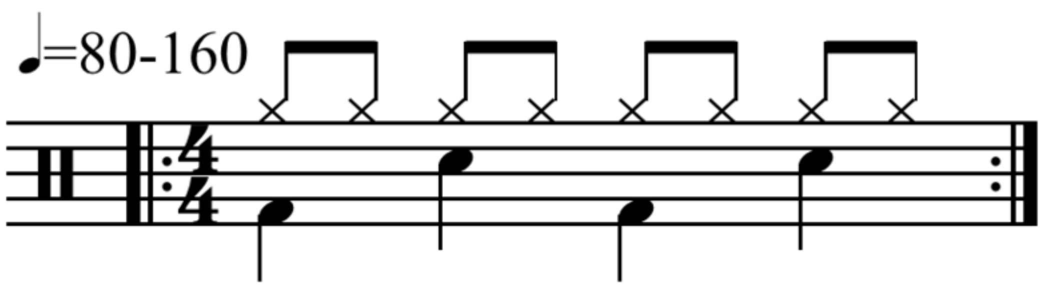


Some Definitions

- Tempo
  - Speed or pace of a given piece, typically measured in beats per minute (BPM)

Some Definitions

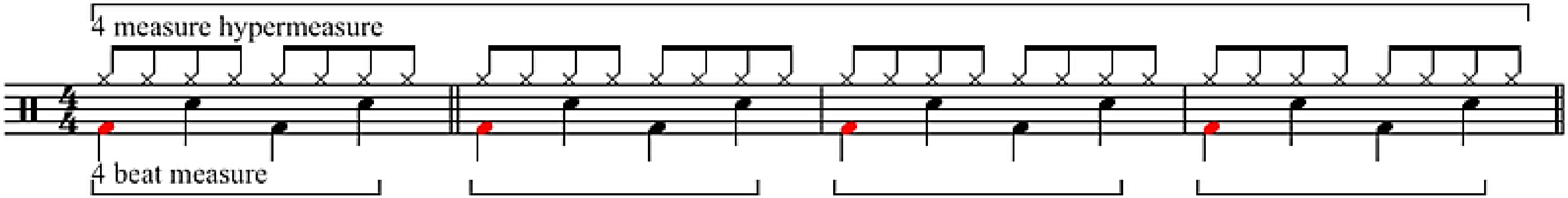
- <u>Bar (or measure)</u>
  - Segment of time defined by a given number of beats



A 4-beat measure drum pattern.
[http://en.wikipedia.org/wiki/Metre_(music)]

Some Definitions

- Meter (or metre)
  - Organization of music into regularly recurring measures of stressed and unstressed beats



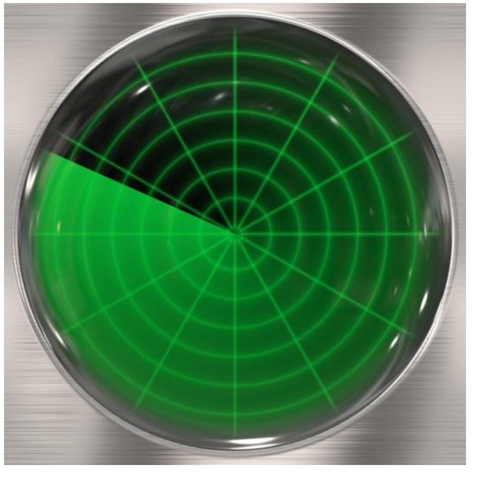
Hypermeter: 4-beat measure and 4-measure hypermeasure. Hyperbeats in red. [http://en.wikipedia.org/wiki/Metre_(music)]

Some Applications

- Onset detection
- Tempo estimation
- Beat tracking
- Higher-level structures

Practical Interest

- Identify/classify/retrieve by rhythmic similarity
- Music segmentation/summarization
- Audio/video synchronization
- And… source separation!

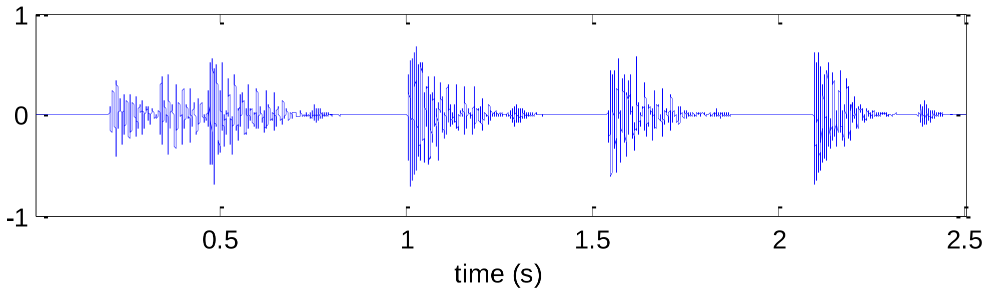


Intellectual Interest

– "Music understanding" [Dannenberg, 1987]
– Music perception
– Music cognition
– And… Fun!

Onset Detection (what?)

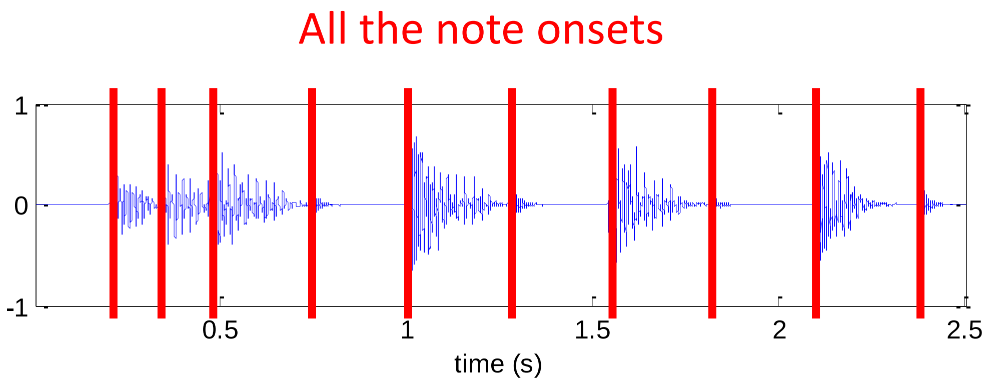
- Identify the starting times of musical elements
- E.g., notes, drum sounds, or any sudden change
- See *novelty curve* [Foote, 2000]



Beginning of *Another one bites the dust* by Queen.

Onset Detection (how?)

– Analyze amplitude (drums have high energy!)
– Analyze other cues (e.g., spectrum, pitch, phase)
– Analyze self-similarity (see *similarity matrix*)



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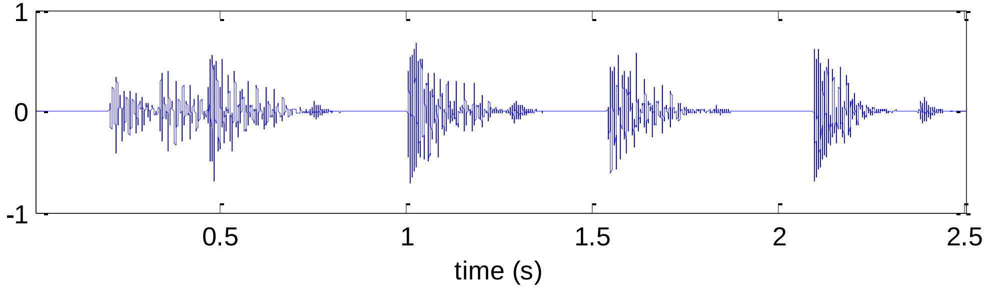
Tempo Estimation (what?)

– Identify periodic or quasi-periodic patterns
– Identify some period of repetition
– See *beat spectrum* [Foote et al., 2001]



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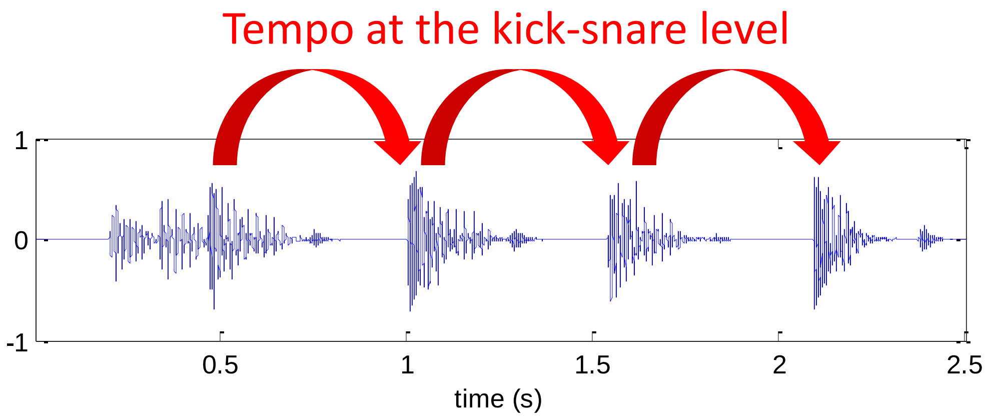
Tempo Estimation (how?)

– Analyze periodicities using the *autocorrelation*
– Compare the onsets with a bank of comb filters
– Use the Short-Time Fourier Transform (STFT)



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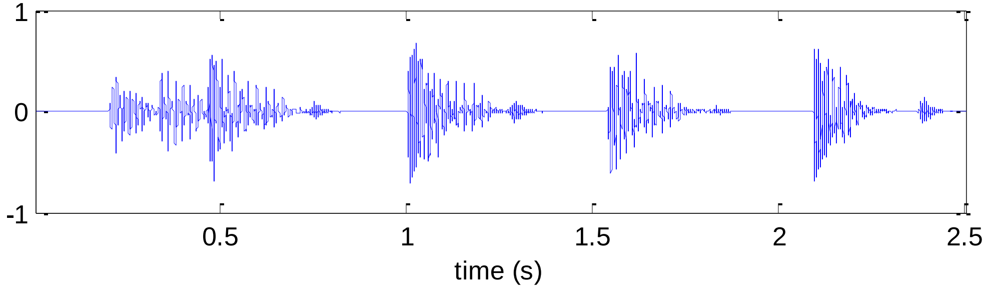
Beat Tracking (what?)

– Identify the beat times
– Identify the times to which we tap our feet
– See (also) *beat spectrum* [Foote et al., 2001]



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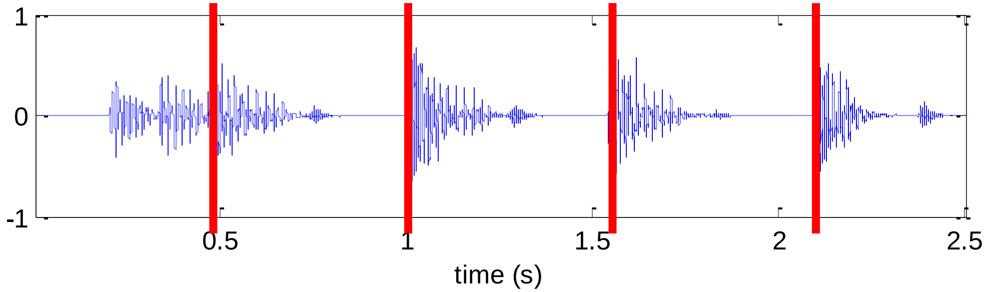
Beat Tracking (how?)

– Find optimal beat times given onsets and tempo
– Use Dynamic Programming [Ellis, 2007]
– Use Multi-Agent System [Goto, 2001]

Beats at the kick-snare level



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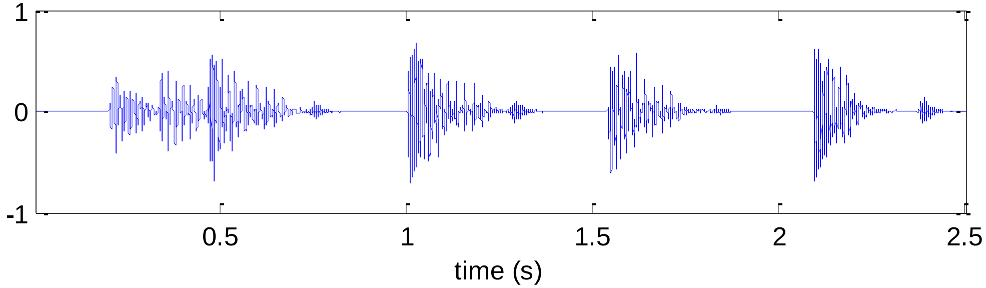
Higher-level Structures (what?)

– Rhythm, meter, etc.
– "Music understanding"
– See (again) *beat spectrum* and *similarity matrix*



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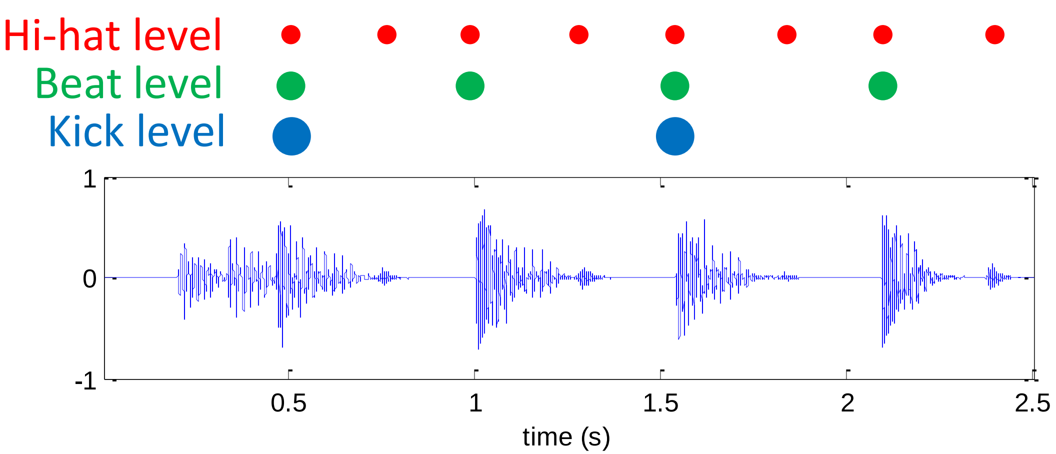
Higher-level Structures (how?)

– Extract onsets, tempo, beat
– Use/assume additional knowledge
– E.g., how many beats per measure? Etc.



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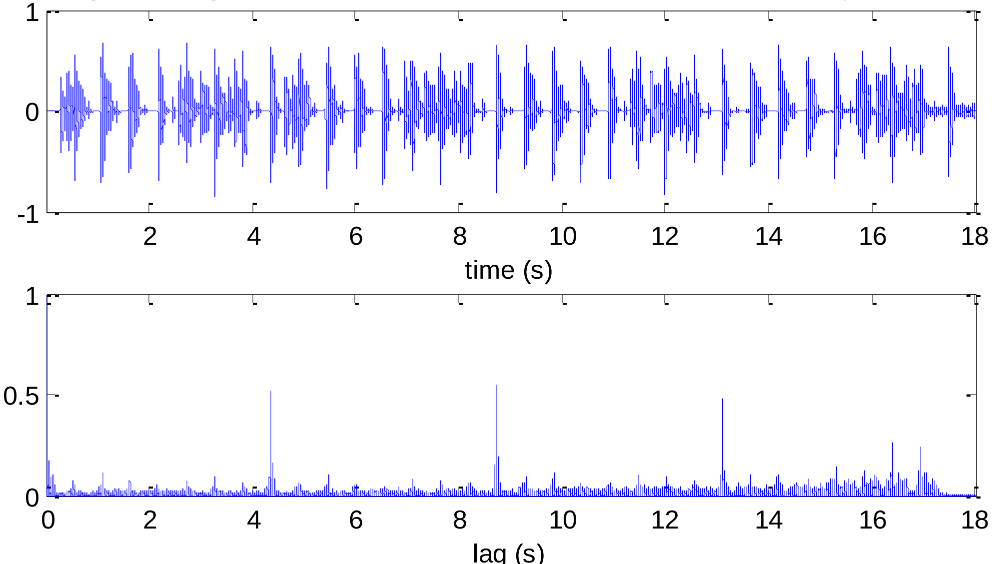
State-of-the-Art

- Some interesting links
  - Dannenberg's articles on beat tracking: http://www.cs.cmu.edu/~rbd/bib-beattrack.html
  - Goto's work on beat tracking: http://staff.aist.go.jp/m.goto/PROJ/bts.html
  - Ellis' Matlab codes for tempo estimation and beat tracking: http://labrosa.ee.columbia.edu/projects/beattrack/
  - MIREX's annual evaluation campaign for Music Information Retrieval (MIR) algorithms, including tasks such as onset detection, tempo extraction, and beat tracking: http://www.music-ir.org/mirex/wiki/MIREX_HOME

The Autocorrelation Function

- Definition
  – Cross-correlation of a signal with itself = measure of self-similarity as a function of the time lag

Beginning of *Another one bites the dust* by Queen.

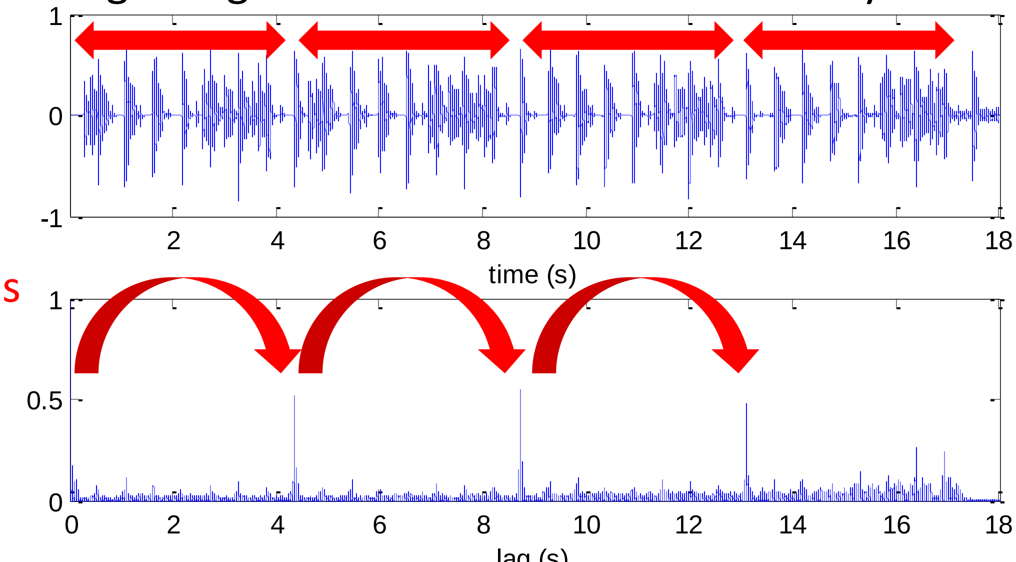


Autocorrelation plot.

The Autocorrelation Function

- Application
  - Identify repeating patterns
  - Identify periodicities

Beginning of *Another one bites the dust* by Queen.



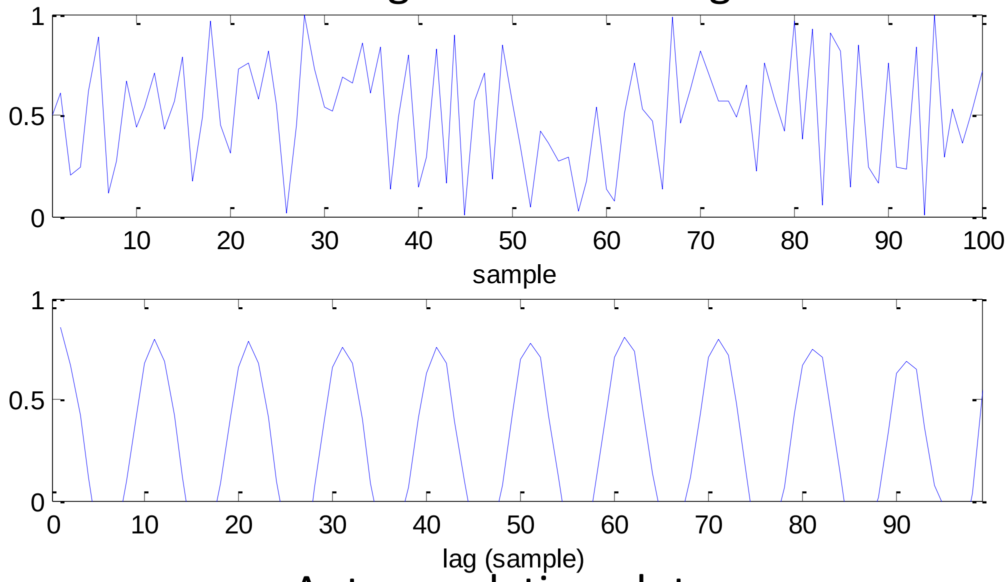
Periodicity of about 4 s

Autocorrelation plot.

The Autocorrelation Function

- Application
  - Identify repeating patterns
  - Identify periodicities

Periodic signal + random signal.

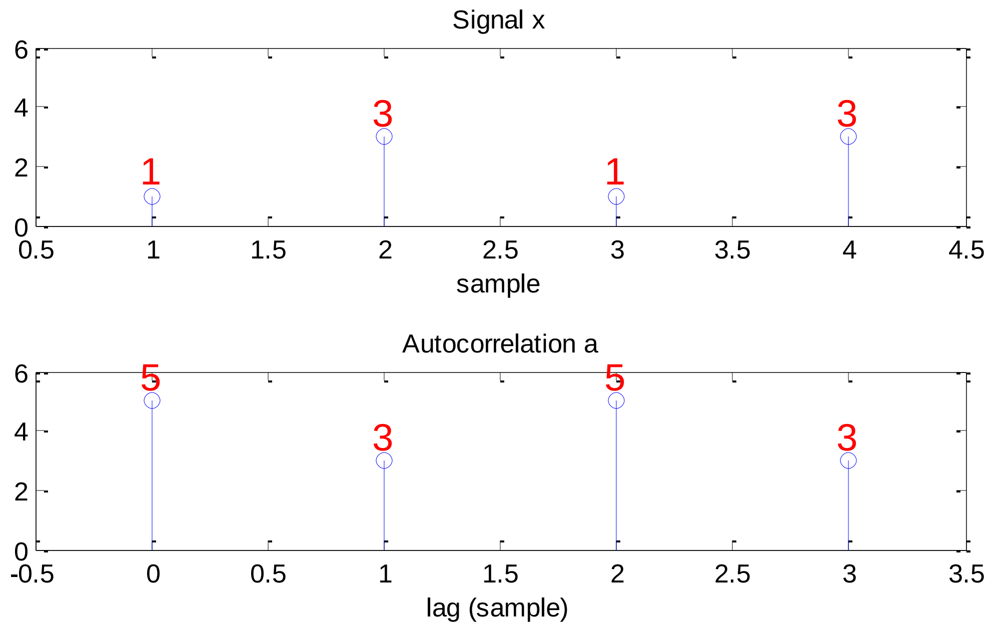


Autocorrelation plot.

The Autocorrelation Function

- Calculation

$$a(j) = \frac{1}{n-j} \sum_{i=1}^{n-j} x(i)x(i+j)$$



The Autocorrelation Function

- Calculation

$$a(j) = \frac{1}{n-j} \sum_{i=1}^{n-j} x(i)x(i+j)$$

samples

	1	2	3	4
$x(i) =$	1	3	1	3
$x(i+0) =$	1	3	1	3

	0	1	2	3
$a(j) =$				

lags

The Autocorrelation Function

- Calculation

$$a(j) = \frac{1}{n-j} \sum_{i=1}^{n-j} x(i)x(i+j)$$

samples

	1	2	3	4
$x(i) =$	1	3	1	3
$x(i+0) =$	1	3	1	3

$$a(j=0) = \frac{1+9+1+9}{4} = 5$$

	0	1	2	3
$a(j) =$	5			

lags

The Autocorrelation Function

- Calculation

$$a(j) = \frac{1}{n-j} \sum_{i=1}^{n-j} x(i)x(i+j)$$

samples

	1	2	3	4
$x(i) =$	1	3	1	3

$x(i+1) =$	1	3	1	3

$a(j) =$	5			
	0	1	2	3

lags

The Autocorrelation Function

- Calculation

$$a(j) = \frac{1}{n-j} \sum_{i=1}^{n-j} x(i)x(i+j)$$

samples

	1	2	3	4	
$x(i) =$		1	3	1	3
$x(i+1) =$	1	3	1	3	

$$a(j=1) = \frac{3+3+3}{3} = 3$$

$a(j) =$	5	3		
	0	1	2	3

lags

The Autocorrelation Function

- Calculation

$$a(j) = \frac{1}{n-j}\sum_{i=1}^{n-j} x(i)x(i+j)$$

samples

	1	*2*	*3*	*4*
$x(i) =$	1	3	1	3

$x(i+2) =$	1	3	1	3

$a(j) =$	5	3		
	0	*1*	*2*	*3*

lags

The Autocorrelation Function

- Calculation

$$a(j) = \frac{1}{n-j} \sum_{i=1}^{n-j} x(i)x(i+j)$$

samples

	1	2	3	4
$x(i) =$	1	3	1	3

$x(i+2) =$	1	3	1	3

$$a(j=2) = \frac{1+9}{2} = 5$$

$a(j) =$	5	3	5	
	0	1	2	3

lags

The Autocorrelation Function

- Calculation

$$a(j) = \frac{1}{n-j} \sum_{i=1}^{n-j} x(i)x(i+j)$$

samples

1 2 3 4

$$x(i) = \boxed{1}\boxed{3}\boxed{1}\boxed{3}$$

$$x(i+3) = \boxed{1}\boxed{3}\boxed{1}\boxed{3}$$

$$a(j) = \boxed{5}\boxed{3}\boxed{5}\boxed{}$$

0 1 2 3

lags

The Autocorrelation Function

- Calculation

$$a(j) = \frac{1}{n-j} \sum_{i=1}^{n-j} x(i)x(i+j)$$

samples

	1	2	3	4
$x(i) =$	1	3	1	3

$x(i+3) =$	1	3	1	3

$$a(j=3) = \frac{3}{1} = 3$$

$a(j) =$	5	3	5	3
	0	1	2	3

lags

The Autocorrelation Function

- Calculation

$$a(j) = \frac{1}{n-j} \sum_{i=1}^{n-j} x(i)x(i+j)$$

samples

	1	*2*	*3*	*4*
$x(i) =$	1	3	1	3

	0	*1*	*2*	*3*
$a(j) =$	5	3	5	3

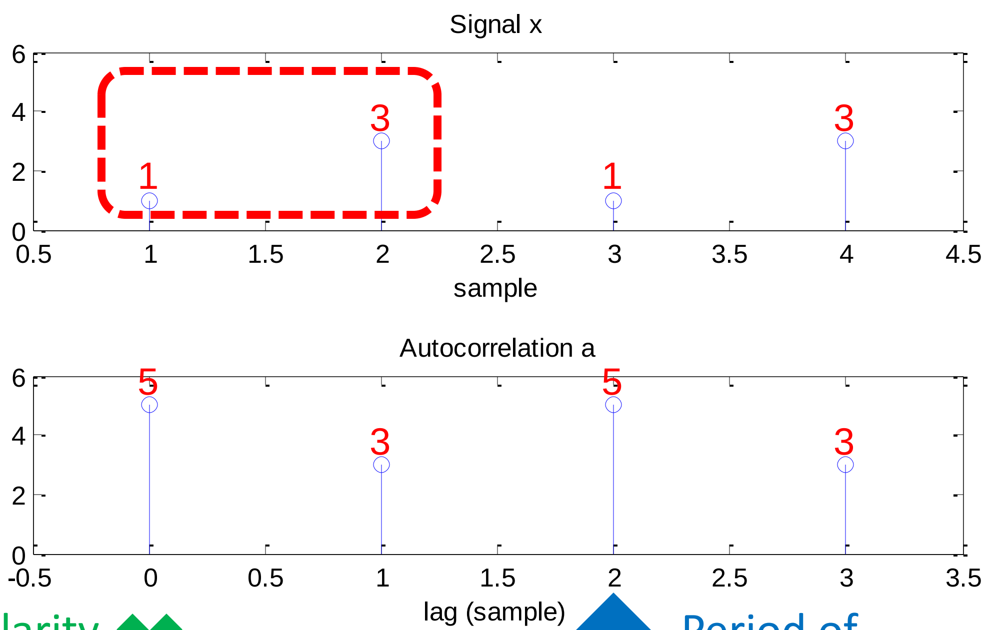
lags

The Autocorrelation Function

- Calculation

$$a(j) = \frac{1}{n-j} \sum_{i=1}^{n-j} x(i)x(i+j)$$

Periodic sequence of 2 samples

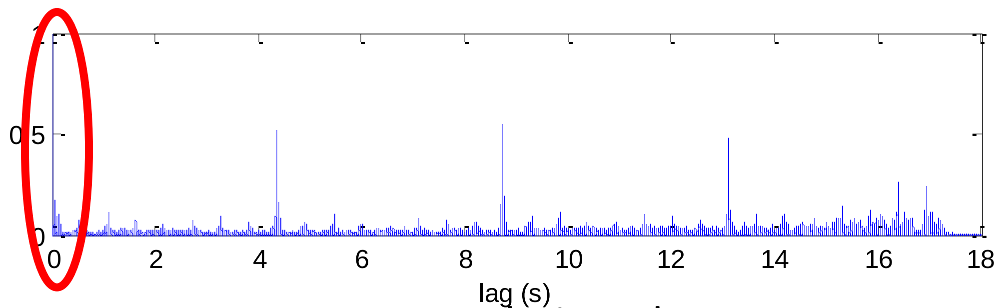


Lag 0 = similarity with itself ✖

Period of 2 samples

The Autocorrelation Function

- Notes
  - The autocorrelation generally starts at lag 0 = similarity of the signal with itself
  - Wiener-Khinchin Theorem: Power Spectral Density = Fourier Transform of autocorrelation

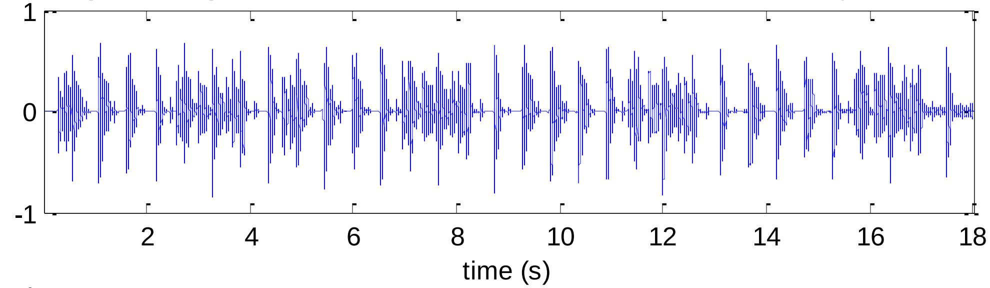


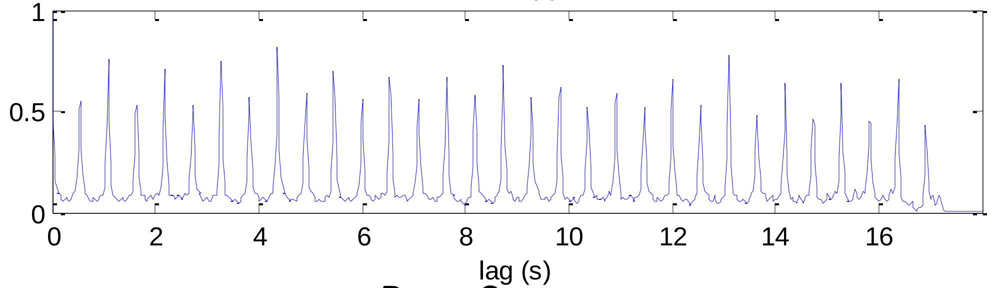
Autocorrelation plot.

Foote's Beat Spectrum

- Definition
  - Using the autocorrelation function, we can derive the beat spectrum [Foote et al., 2001]

Beginning of *Another one bites the dust* by Queen.



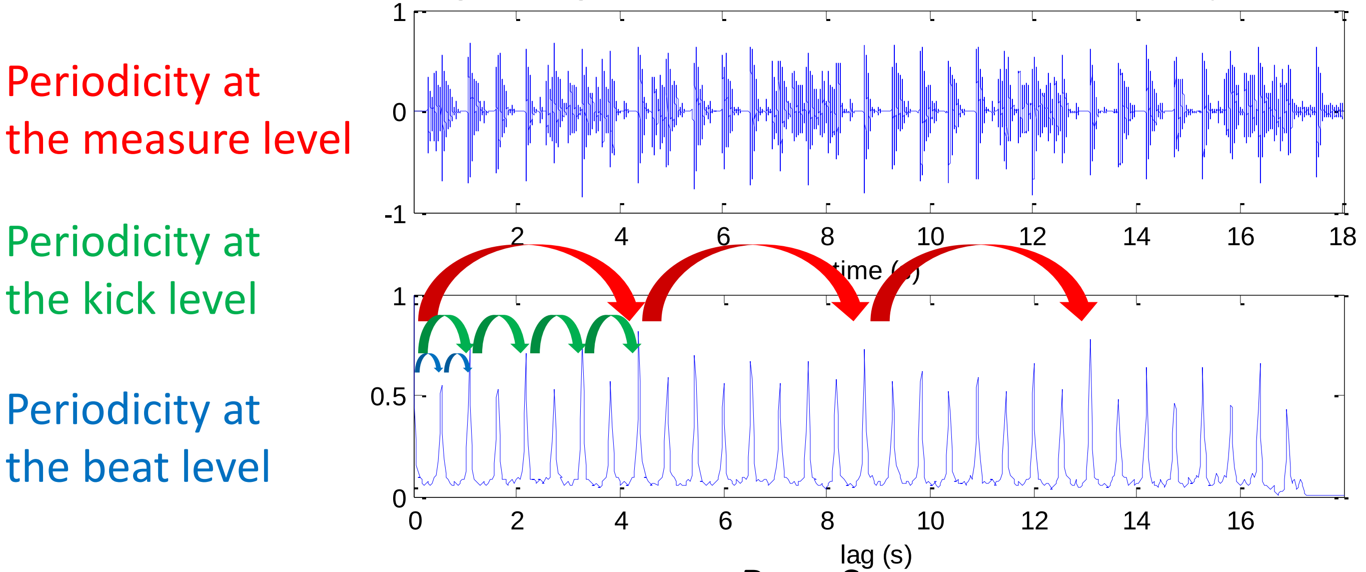


Beat Spectrum.

Foote's Beat Spectrum

- Application
  - The beat spectrum reveals the hierarchically periodically repeating structure

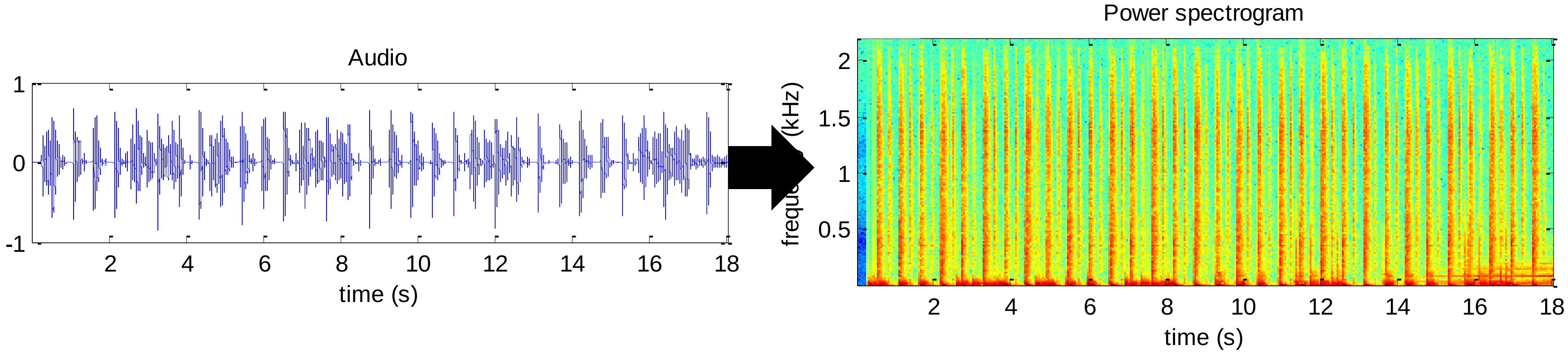
Beginning of *Another one bites the dust* by Queen.



Beat Spectrum.

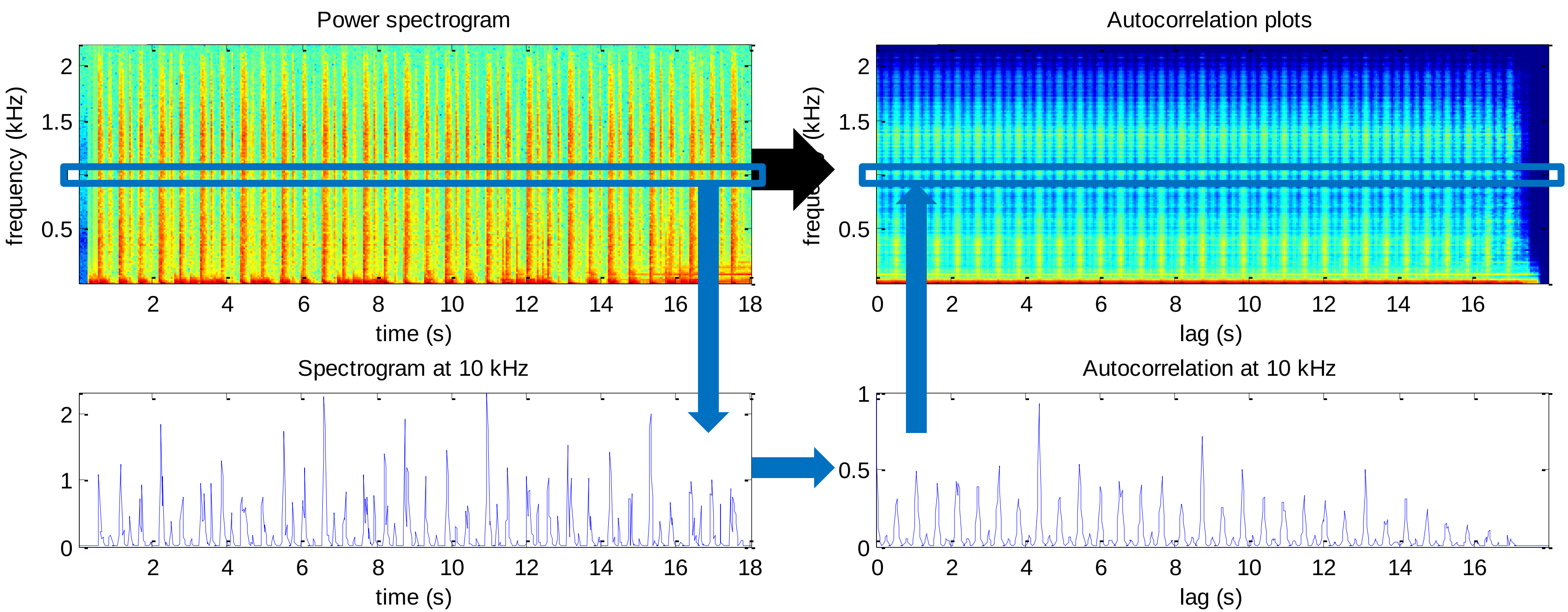
Foote's Beat Spectrum

- Calculation
  - Compute the power spectrogram from the audio using the STFT (square of magnitude spectrogram)



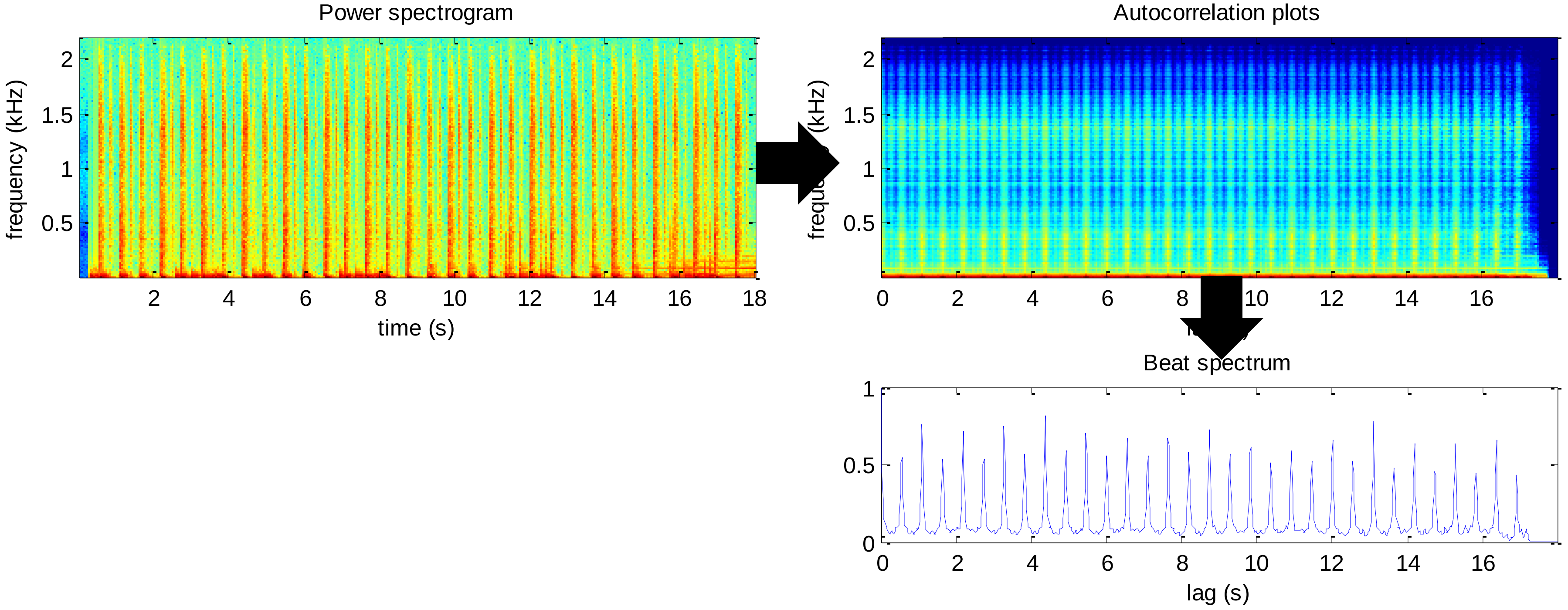
Foote's Beat Spectrum

- Calculation
  - Compute the autocorrelation of the rows (i.e., the frequency channels) of the spectrogram



Foote's Beat Spectrum

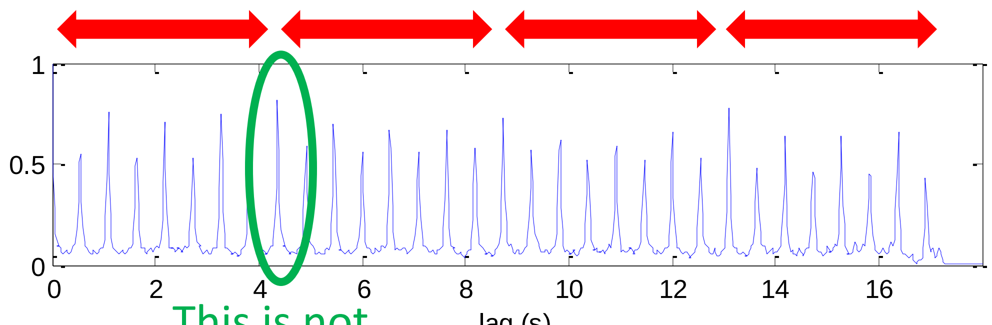
- Calculation
  - Compute the mean of the autocorrelations (of the rows)



Foote's Beat Spectrum

- Notes
  - The first highest peak in the beat spectrum does not always correspond to the repeating period!
  - The beat spectrum does not indicate where the beats are or when a measure starts!

This is how you find the period…



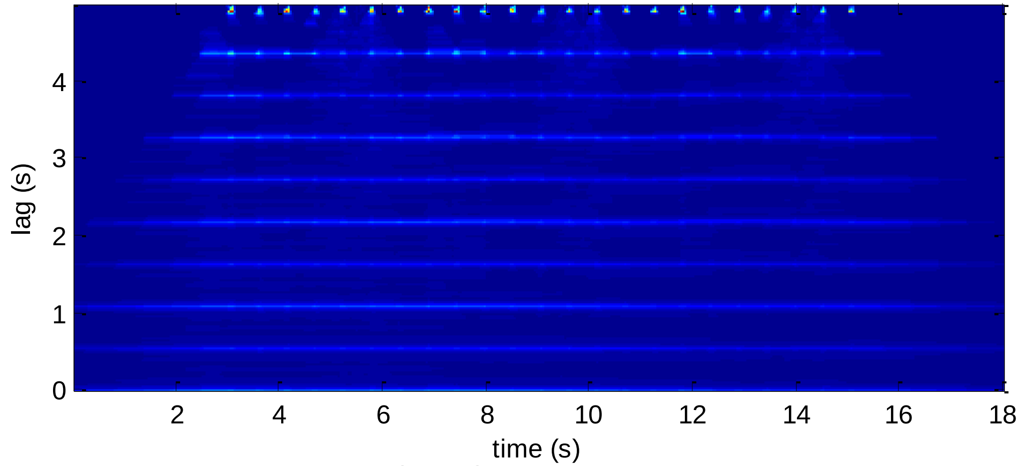
This is not the period

Beat Spectrum.

Foote’s Beat Spectrum

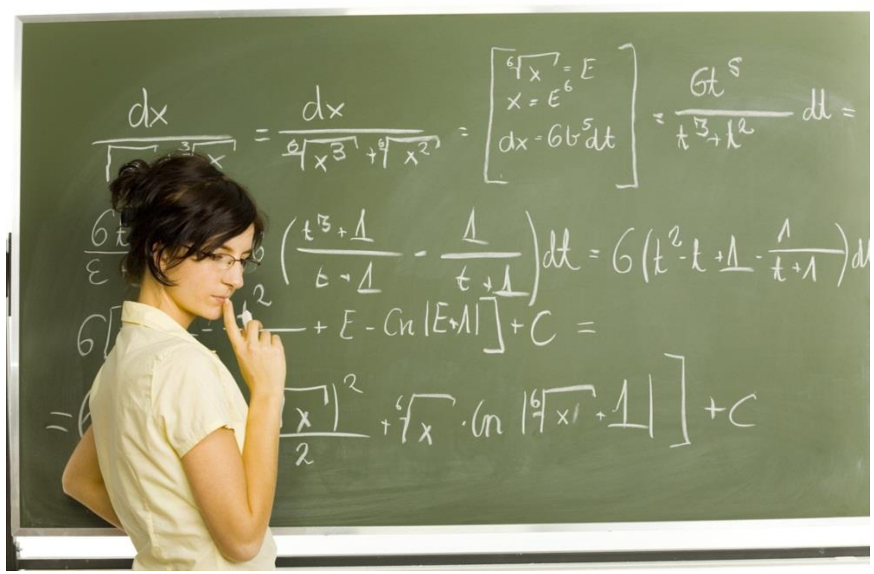
- Notes
  - The beat spectrum can also be calculated using the *similarity matrix* [Foote et al., 2001]
  - A *beat spectrogram* can also be calculated using successive beat spectra [Foote et al., 2001]

Beat Spectrogram.



Foote's Beat Spectrum

- Question
  - Can we use the beat spectrum for source separation?...
  - To be continued…



References

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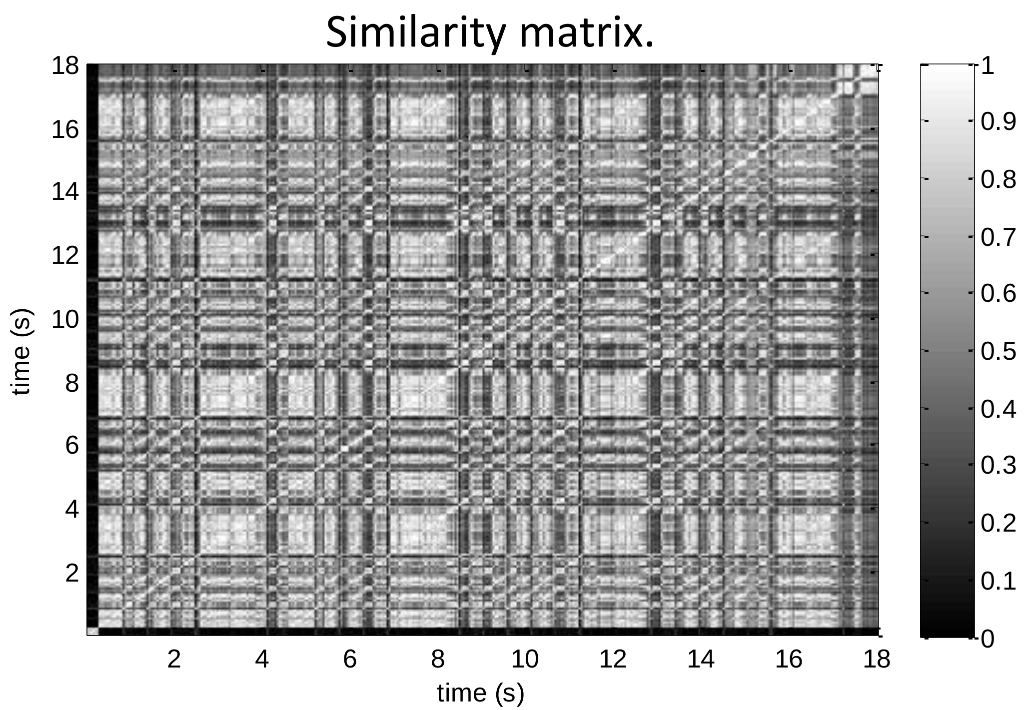
- Wikipedia, “Meter,” http://en.wikipedia.org/wiki/Metre_(music), 2012.

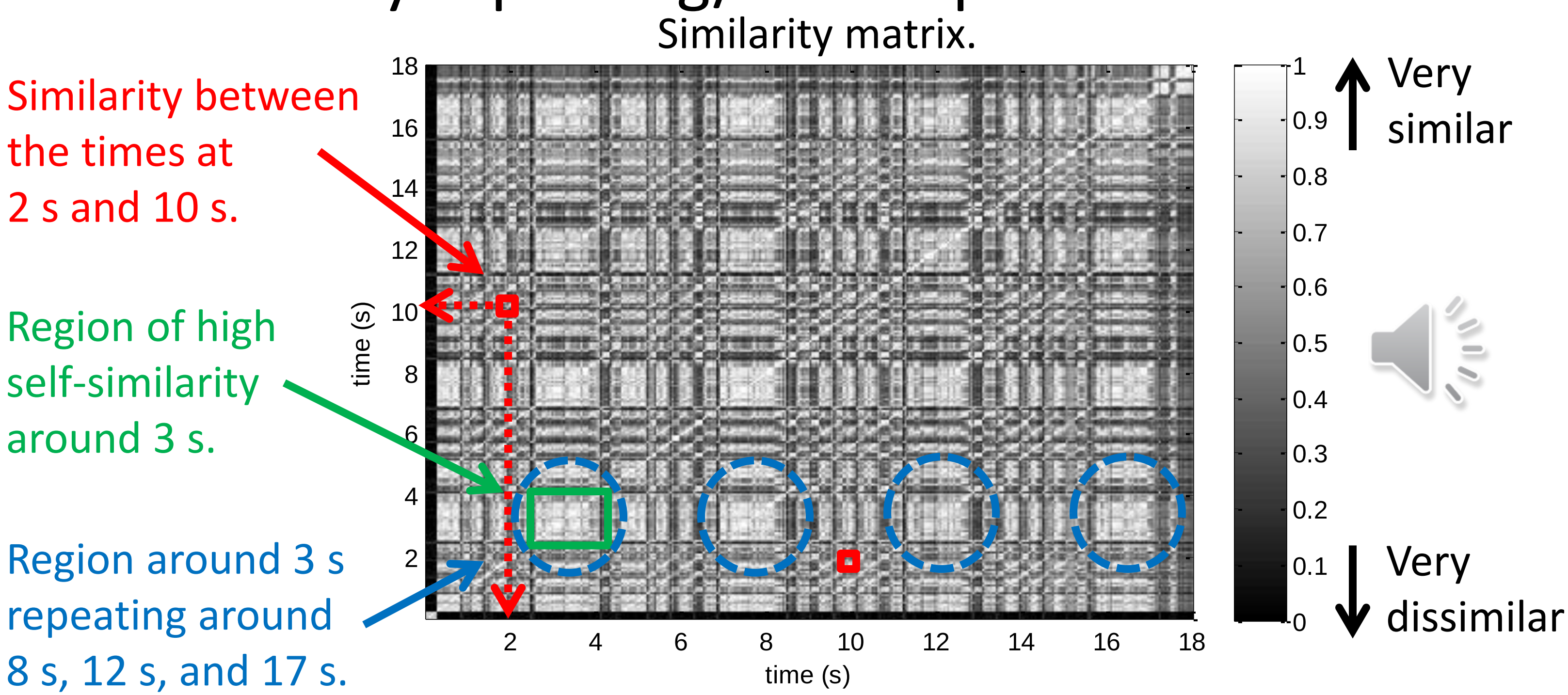
The Similarity Matrix

- Definition
  - Matrix where each point measures the similarity between any two elements of a given sequence



The Similarity Matrix

- Application
  – Visualize time structure [Foote, 1999]
  – Identify repeating/similar patterns



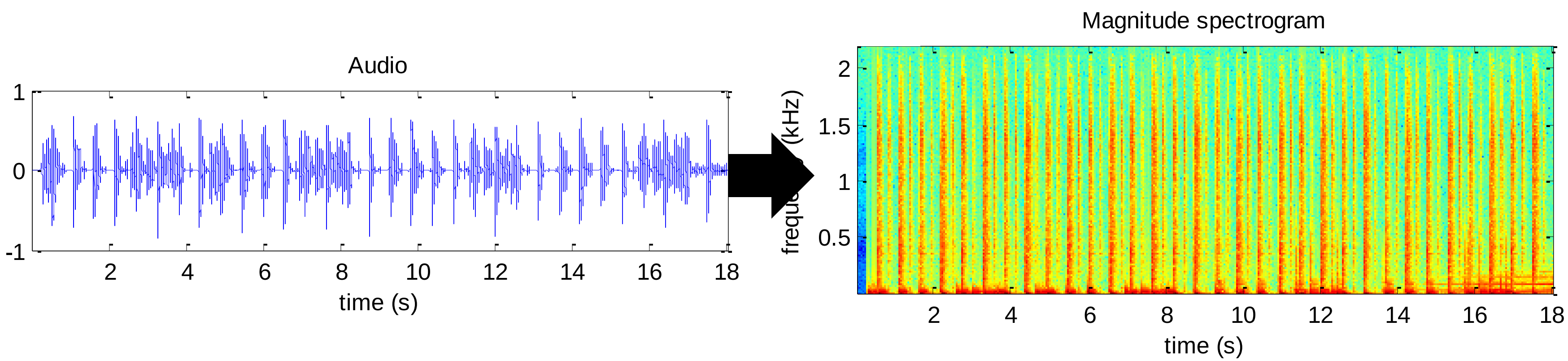
The Similarity Matrix

- Calculation
  - The similarity matrix S of X is basically the matrix multiplication between transposed X and X, after (generally) normalization of the columns of X

$$S(j_1, j_2) = \frac{\sum_{k=1}^{n} X(k,j_1)X(k,j_2)}{\sqrt{\sum_{k=1}^{n} X(k,j_1)^2}\sqrt{\sum_{k=1}^{n} X(k,j_2)^2}}$$

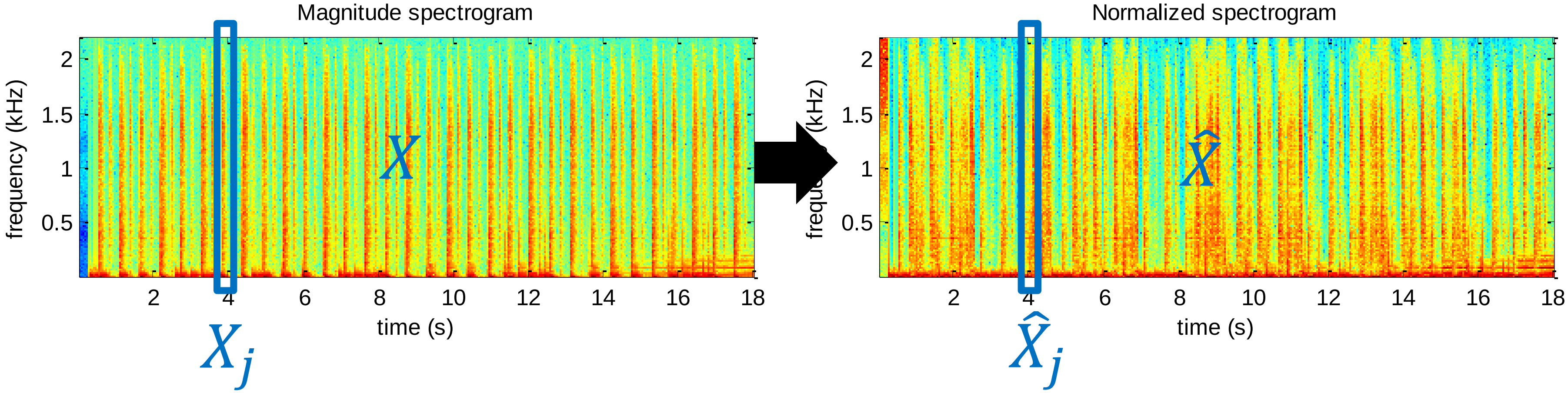
The Similarity Matrix

- Calculation
  - Compute the magnitude spectrogram from the audio using the STFT



The Similarity Matrix

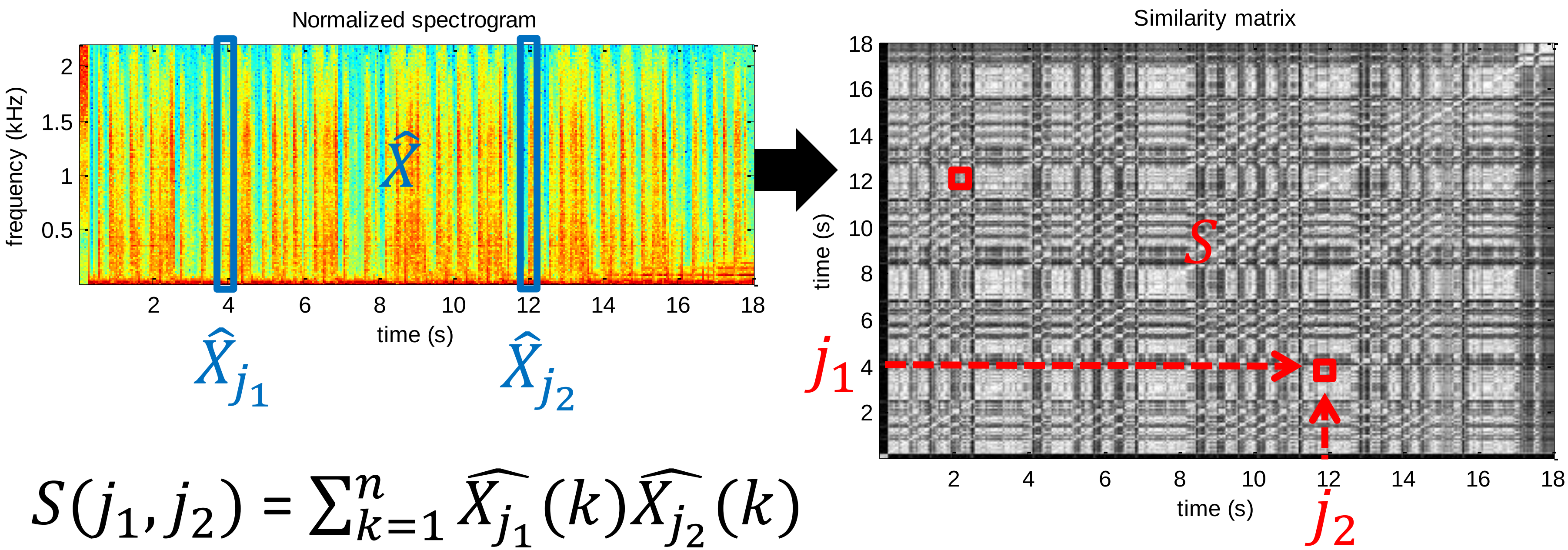
- Calculation
  - Normalize the columns of the spectrogram by dividing them by their Euclidean norm



$$\hat{X}_j(i) = \frac{X_j(i)}{\sqrt{\sum_{k=1}^{n} X_j(k)^2}}$$

The Similarity Matrix

- Calculation
  - Compute the dot product between any two pairs of columns and save them in the similarity matrix



$$S(j_1, j_2) = \sum_{k=1}^{n} \widehat{X_{j_1}}(k)\widehat{X_{j_2}}(k)$$

The Similarity Matrix

- Notes
  - The similarity matrix can also be built from other features (e.g., MFCCs, chromagram, pitch contour)
  - The similarity matrix can also be built using other measures (e.g., Euclidean distance)