Topic

Convolution and Correlation

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Convolution

- **convolution** is a mathematical operator which takes two functions $x$ and $h$ and produces a third function that represents the amount of overlap between $h$ and a reversed and translated version of $x$.

- In signal processing, one of the functions $(h)$ is taken to be a fixed filter *impulse response*, and the other $(x)$ the input signal.

\[
(h \ast x)(t) \equiv \int_{-\infty}^{\infty} h(\tau)x(t - \tau)\,d\tau
\]
Discrete Convolution

- **convolution** is a mathematical operator which takes two functions $x$ and $h$ and produces a third function that represents the amount of overlap between $h$ and a reversed and translated version of $x$.
- In signal processing, one of the functions ($h$) is taken to be a fixed filter *impulse response*, and the other ($x$) the input signal.

\[(h \ast x)[n] \equiv \sum_{k=-\infty}^{\infty} h[k]x[n-k]\]
import numpy as np

def convolution(A,B):
    lengthA=np.size(A)
    lengthB=np.size(B)

    C = np.zeros(lengthA + lengthB -1)

    for m in np.arange(lengthA):
        for n in np.arange(lengthB):
            C[m+n] = C[m+n] + A[m]*B[n]

    return C
Let’s look at Convolution
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Let’s look at Convolution

![Graphs showing convolution of f[n] and g[n]]
Let’s look at Convolution
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![Convolution Diagram](image-url)
Let’s look at Convolution

Let’s look at Convolution

![Convolution Diagram]
Let’s look at Convolution
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Let’s look at Convolution

\[ f[n] \]

\[ g[n] \]

\[ f[n] \ast g[n] \]

\[ f[n] \ast g[n] \]
Let’s look at Convolution

\[ f[n] \]

\[ g[n] \]

\[ f[n] * g[n] \]

\[ f[n] * g[n] \]
Let’s look at Convolution

![Convolution Diagram](image)
Let’s look at Convolution

Let's examine the convolution of two functions, $f[n]$ and $g[n]$. The convolution of $f[n]$ and $g[n]$ is shown in the third graph, $f[n-n_0] * g[n_0]$. The result of the convolution, $f[n] * g[n]$, is illustrated in the fourth graph.
Let’s look at Convolution
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Let’s look at Convolution
Cross-correlation

- **Cross-correlation** is a measure of similarity of two functions at time-lag $t$ applied to one of them. It is a LOT like convolution…

$$\left( h \quad \text{apple} \quad x \right)(t) \equiv \int_{-\infty}^{\infty} h^*(\tau) x(t + \tau) \, d\tau$$

Cross-correlation operator
Should be a star
Couldn’t find “star” in my font

Means “complex conjugate of $h$”
• Convolution

\[(h \ast x)(t) \equiv \int_{-\infty}^{\infty} h(\tau) x(t - \tau) \, d\tau\]

• Cross-correlation

\[(h \bigcirc x)(t) \equiv \int_{-\infty}^{\infty} h^*(\tau) x(t + \tau) \, d\tau\]
Cross-correlation in Python Code

We can easily implement cross correlation with convolution as follows:

```python
def crosscorrelation(A,B):
    return convolution(np.conj(A),B[::-1])
```

Better yet, use the built in Python functions…

```python
np.convolve(A,B,"full")    # for convolution
np.correlate(A,B,"full")   # for cross correlation
```
**Auto-correlation**

- **Auto-correlation** is a measure of similarity of a function to itself at time-lag $t$. It is a special case of cross-correlation (cross-correlation of a function with itself).

\[
(x \star x)(t) \equiv \int_{-\infty}^{\infty} x^*(\tau)x(t+\tau) \, d\tau
\]

Means “complex conjugate of $f$”

Cross correlation
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Relating them all

Convolution

Cross-correlation

Autocorrelation

Image from http://commons.wikimedia.org/wiki/File:Comparison_convolution_correlation.svg
Convolution and Fourier transform

• An important property of the Fourier transform: converts convolution in the time domain into multiplication in the frequency domain.

Convolution...

\[ y(t) = h(t) * x(t) \]

In the time domain:

\[ y(t) = \int h(\tau)x(t - \tau) \, d\tau \]

In frequency domain:

\[ Y(\omega) = H(\omega)X(\omega) \]