# CS395/495: IBMR Take-Home Final Exam

Assigned: 3:30pm,Thurs June 5, 2003
Due: **<u>no later than</u>** 3:30pm,Thurs June 12, 2003

Turn in your work either on paper brought to my office, or by e-mail to the both TA and the instructor (abhinav @cs.northwestern.edu, jet@cs.northwestern.edu). Be sure to number the pages and put your name on each page. Please do your own work—do not discuss the problems or your solutions with other students until after the due date.

1) **$P^3$ Planes:**
(a) Find the 4-vector describes the plane in $P^3$ that passes through these three Cartesian $R^3$ points; the x axis at (xp,0,0), the y axis at (0, yp,0) and the z axis at (0,0,zp) .
(b) Write an H matrix that will transform this plane to the x-y plane.
(c) Find the direction vector (a 4-vector) D that is perpendicular to the plane given in (a).
(d) Construct a plane that includes the Cartesian R3 point [a,b,c] and the line described by the line-span matrix
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W* = [d e f g]
     [h j m n]
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2) **$P^3$ Lines:**
Consider a line through two points $[a,b,c]^T$ and $[d,e,f]^T$ in Cartesian $R^3$ space:
(a) Write a parametric equation for a line L(t) in $P^3$ space that passes through the two $R^3$ points, where t measures distance along the line, where $L(0) = [a,b,c,1]^T$, and t is >0 at the 'd,e,f' point.
(b) Write this $P^3$ line as a point-span matrix W:
(c) Write this $P^3$ line as a line-span matrix W*:
(d) Write the line as a Plucker matrix P:
(e) Find the expression for the point at the intersection of this line with the $P^3$ plane given by $[g,h,j,k]^T$

3) **P3 Lines:** Why are Plucker matrices useful? Give an example where a Plucker Matrix formulation of lines is a better choice than a simple span W or W*.

4) **Constraints**: In $P^3$ space, what are the number of degrees of freedom (e.g. what is the rank) of:
a) a point?
b) an ideal point?
c) a line?
d) a plane?
e) the plane at infinity?
f) a direction?
g) a camera?
h) a pair of cameras bolted together on the same frame?
Be sure to justify each answer. Single-number answers will receive only half credit; for full credit you must explain why you chose that number.

5) **Infinities:** In $P^3$ space, show that all lines include an ideal point, and prove that all lines intersect the plane at infinity.

6) **Conics in $P^3$ to measure angles**: Consider two P3 planes named p1 and p2. After projective transformation by the 4x4 matrix H, these planes become p1' and p2' respectively. We don't know p1, p2, or the H matrix, but we *do* know the transformed absolute dual quadric $Q^*_\infty{}'$ and the transformed plane p1'. We also know that the plane p1 is perpendicular to p2.
Show how to find p2' using equations and the information we have.

7) **$P^3$ Camera**: Let integer values of (x,y) in the image plane of a camera define pixel, and construct a camera matrix P that meets these requirements:
--the field-of-view is 48 degrees horizontally, 36 degrees vertically;
--the image has 1280 pixels horizontally and 1024 scanlines vertically, with pixel (0,0) at the lower-left corner of the image, and the principal point at (640,512).
--the image has no affine distortion,
--the camera is located at (a,b,c) in R3 Cartesian coordinates. The principal point is an image of the point at (d,e,f) in Cartesian coordinates, and the P3 direction $[0,1,0,0]^T$ (e.g. the world-space y-axis direction) is always transformed to an image line parallel to the image's y axis.

8) **Epipolar Geometry :**
a) Construct a pair of camera matrices P0, P1. Matrix P0 is the basic camera $P_0$ from class lecture notes after it is translated to [0,0,-1] in Cartesian R3 space. Matrix P1 is a companion camera, made by moving an exact copy of the P0 camera to place its camera center at (+2, +1, 0) in $R^3$ . The P1 camera was then rotated, first about its y axis so that the principal axis intersects the world-space y axis, and then about the camera's x axis so that the principle axis intersects the origin.
b) Find the fundamental matrix F for this pair of cameras.
c) Both cameras form an image of an ordinary sharpened pencil. In the 2D image from camera P0, we found the tip of the pencil is at point x0 = [0.5, 0.5]. Find the corresponding epipolar line in the image from the P1 camera; (write it as a P2 line).
d) Along the epipolar line in the image from camera P1, we searched and found the tip of the pencil again at point x1; by chance, it was the point closest to the principal point of the image. Compute x1.
e) From the image-space points x0 and x1, compute the 3D position X of the pencil point.

9) **Photometry:** Write the BRDF function of a purely diffuse gray material that re-radiates exactly half of all the incident light energy it receives, and radiates that light equally in all directions. Be sure to express your answer in the proper units (e.g. 1/steradians).

10) **Discussion/Opinion Question:** In the EGRW2001 paper "Polyhedral Visual Hulls for Real-Time Rendering" (Week 9 reading assignment; handed out & on website) Matusik et al. show how to make each 3D polygon of a visual hull by 2D projection and 2D clipping. (They also describe a way to accelerate this process by 'edge bins', but ignore that for this question).
    a) give a step-by-step description of this method, sufficient to implement it in Project D.
    b) Discuss the advantages and difficulties you might encounter if this method were used as a replacement for Project D.