Structure Learning

Road Map

- Basics of Probability and Statistical Estimation
- Bayesian Networks
- Markov Networks
- Inference
- Learning
  - Parameters, **Structure**, EM
- HMMs

Learning Structure

- Hard problem
  - Finding the BN structure with the highest "score" among those structures with at most *k* parents is NP hard for $k>1$ (Chickering, 1995)
- Inputs
  - Data (potentially incomplete)
- Outputs
  - Graphical model structure (we'll focus on Bayes Nets)
- Approaches
  - Constraint-based
  - Score-based approaches
    - Local search
  - Bayesian Model Averaging

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Constraint-based Approaches

- Idea: we know how to construct a Bayes Net if we can perform **independence tests**
  - (**A** $\perp$ **B** | **C**) ?
- Naïve construction
  - depends on variable ordering
  - Issues potentially large independence queries
- A more sophisticated PDAG construction process works better (see book)

Constraint-based approach guarantees

- Can uncover a *perfect* map using a polynomial # of tests if:
  - Bounded in-degree *d* in *G** (true graph)
  - Perfect independence queries up to size 2*d* + 2
    (Strong)
  - P* (true dist.) is *faithful* to G*
    (Also strong)
    - i.e., any independencies in P* reflected as d-separation in G*

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Scoring Structures

- Maximum likelihood *G*
  - Choose $G = \arg\max_G \max_\theta P(\text{Data} \mid \theta)$
- Or MAP:
  - Choose $G = \arg\max_G \max_\theta P(\text{Data} \mid \theta)\, P(\theta)$
- …what’s wrong with these?

Bayesian Score

- Bayesian Score for G =

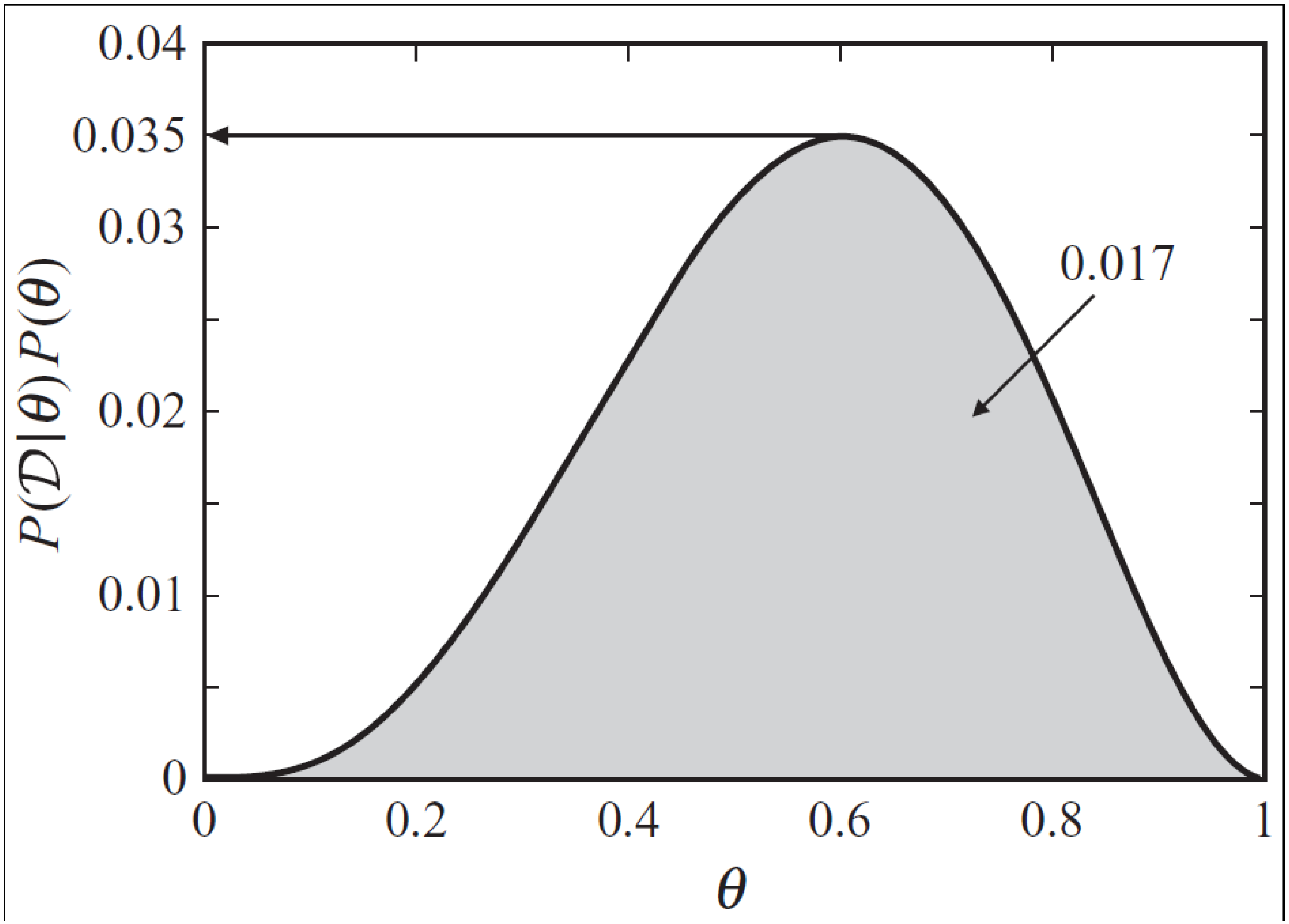
  prior for G

  +

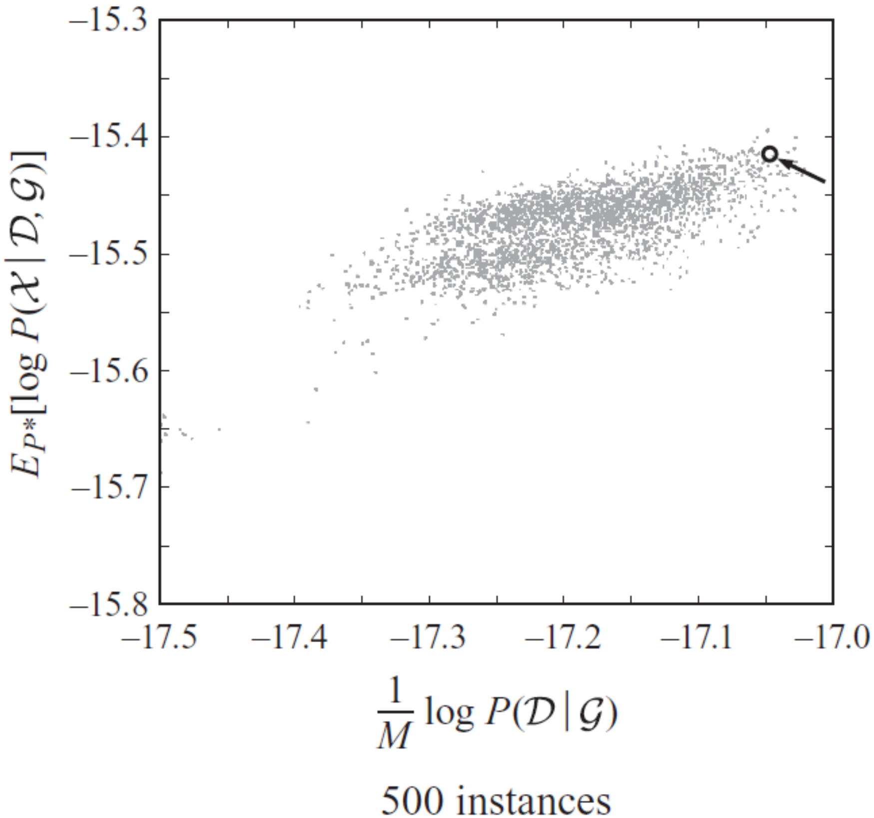
  likelihood integrated over all parameters for G

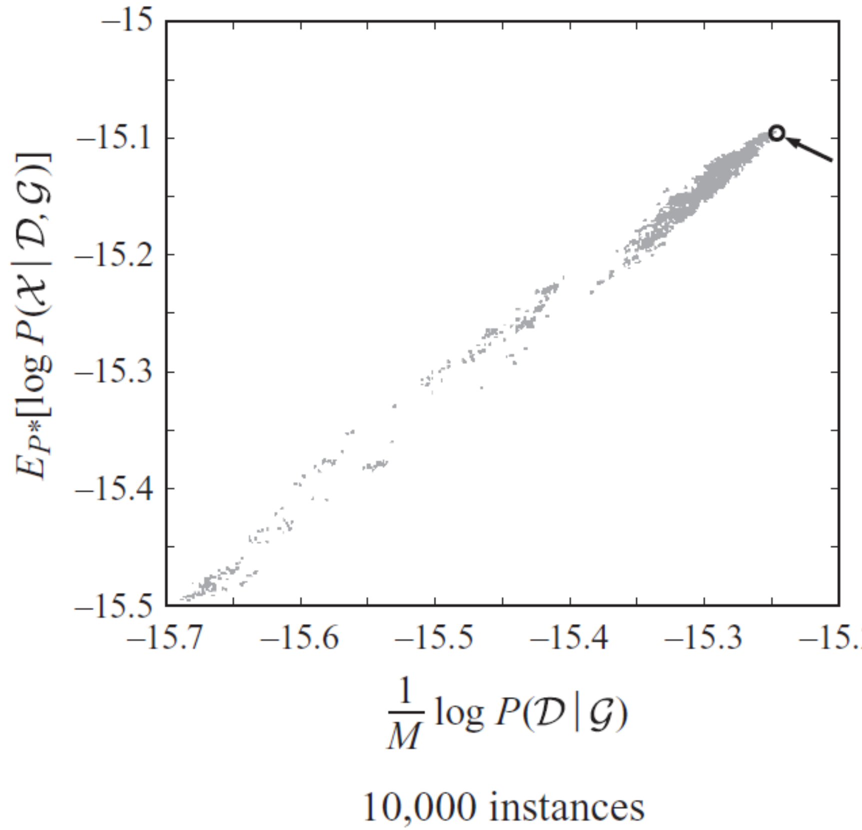
- BayesianScore(G : Data) = log P(Data | G) + log P(G)
- P(Data | G) = $\int_{\Theta_G}$ P(Data | θ_G, G) P(θ_G | G) dθ_G

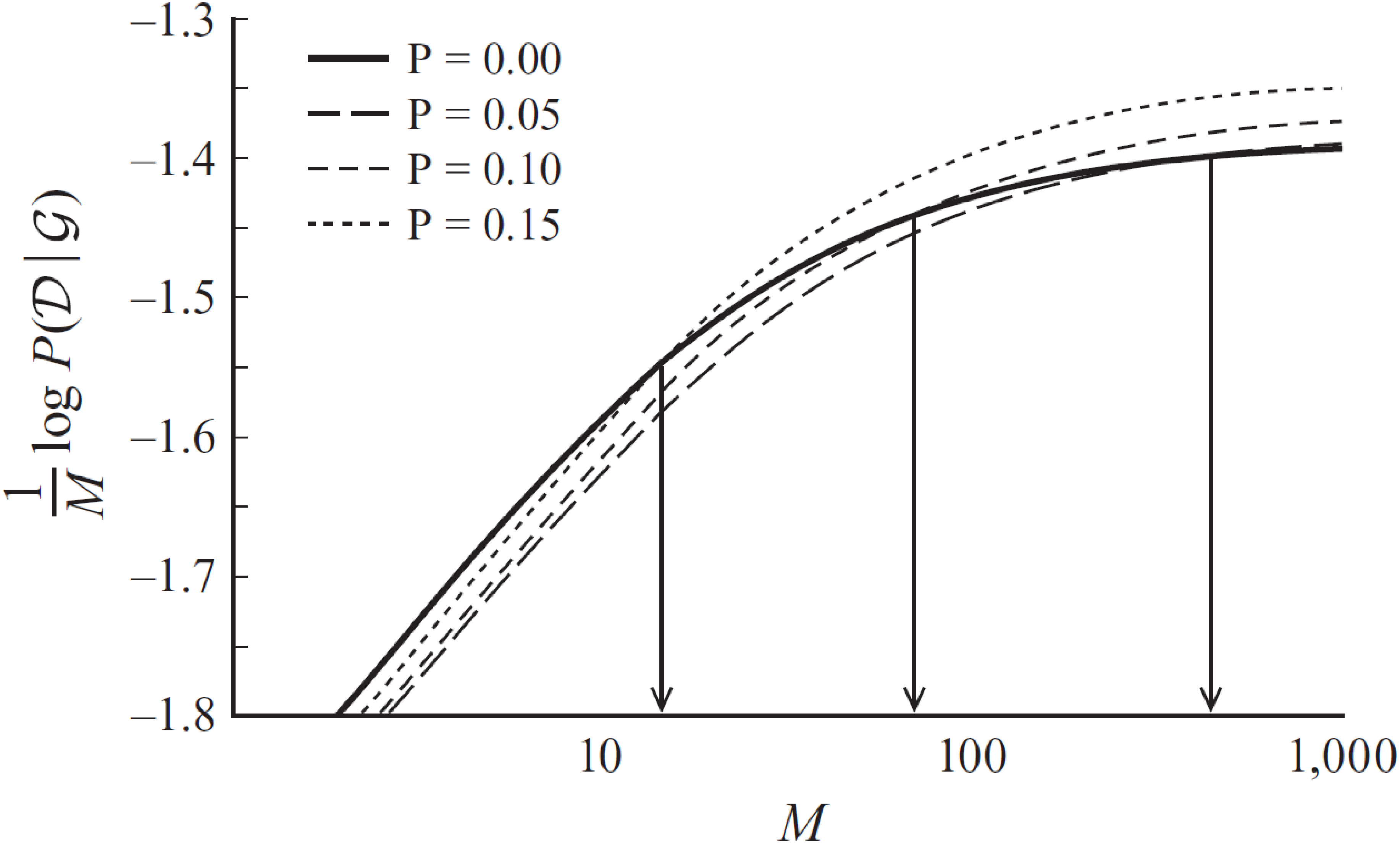
Integrating over parameters



Training (x-axis) vs. Test (y-axis) Perf.







P = 0.00
P = 0.05
P = 0.10
P = 0.15
−1.3
−1.4
−1.5
−1.6
−1.7
−1.8
$\frac{1}{M} \log P(\mathcal{D} \mid \mathcal{G})$
10
100
1,000
M

Bayesian Information Criterion

- Bayes Score includes:
  - $P(\text{Data} \mid G) = \int_{\Theta_G} P(\text{Data} \mid \theta_G, G)\, P(\theta_G \mid G)\, d\theta_G$

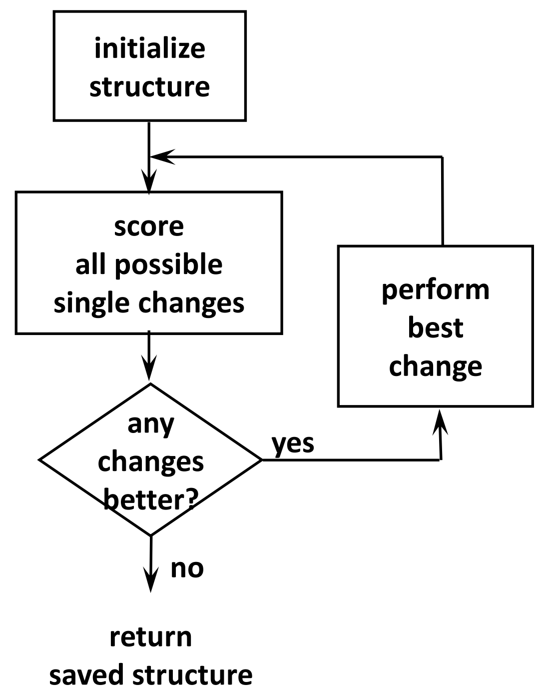
- Integral sometimes difficult

- Approximation:

  $$\text{score}_{\text{BIC}}(G) = -(\text{Dim}[G]/2) \log M + \log \max_{\theta_G} P(\text{Data} \mid \theta_G)$$

Structure search

- Finding the BN structure with the highest score among those structures with at most *k* parents is NP hard for *k*>1 (Chickering, 1995)
- Heuristic methods
  – Greedy
  – Greedy with restarts



Structure priors

- Lots of options
  - All possible structures equally likely
  - Partial ordering, required / prohibited arcs
  - Prior(G) α Similarity(G, Gprior)

Learning Structure

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Bayesian Model Averaging

- Previous methods all find a single graph *G*
- *Bayesian model averaging* instead makes predictions by averaging over structures:

  P(test example | Data) =
  $\sum_G$ P(test example | Data, G) P(G | Data)