Markov Networks

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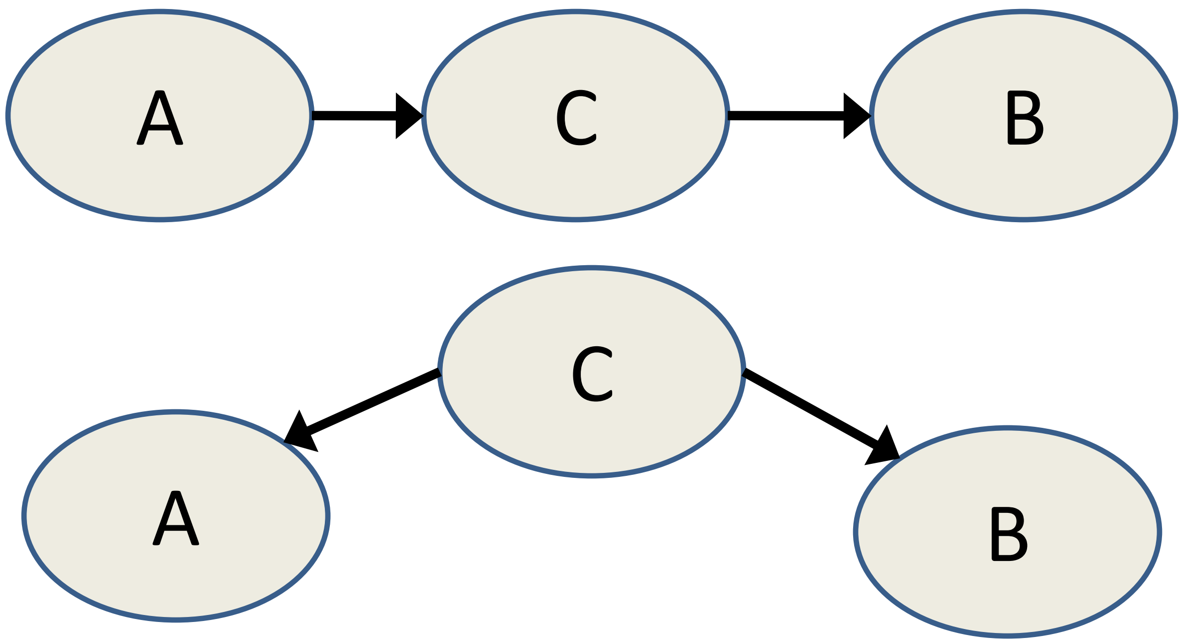
Northwestern EECS 395/495 Fall 2013

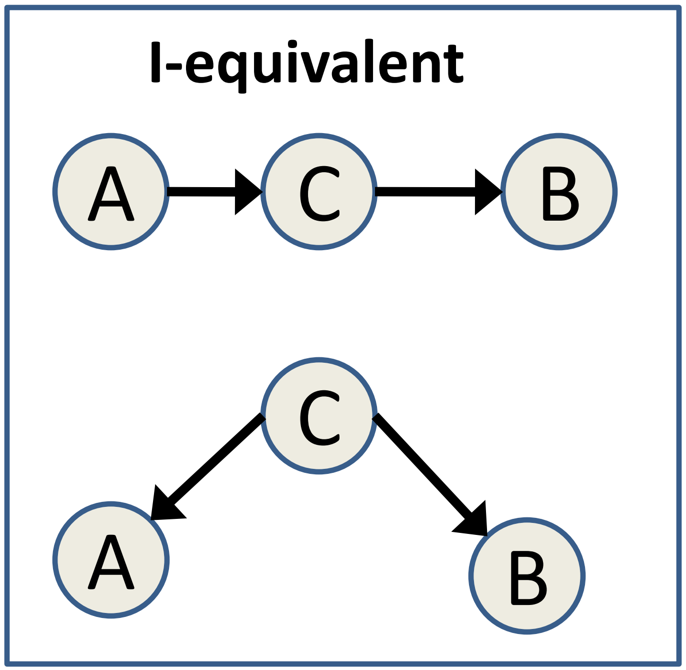
First: Perfect Maps and I-Equivalence

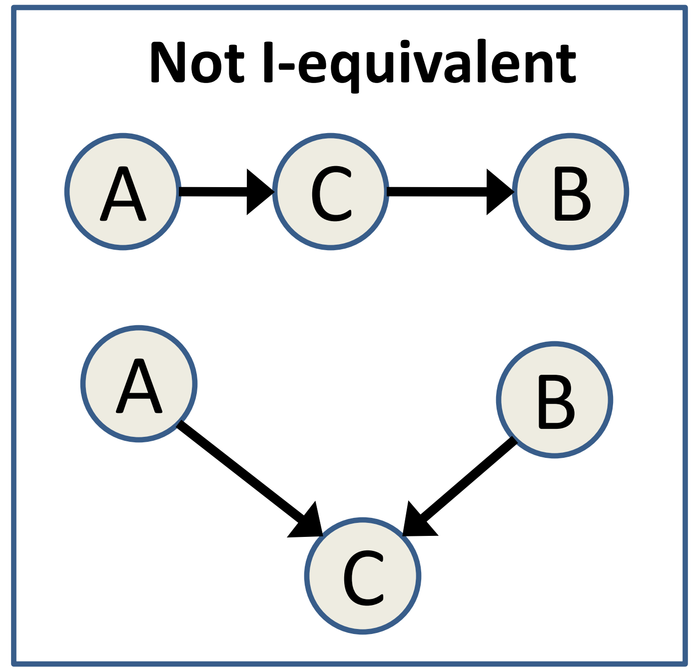
- ***Perfect Map for S:*** A graph for a set *S* of *independence assertions*, i.e. statements of the form ($\mathbf{X} \perp \mathbf{Y} \mid \mathbf{Z}$)
- E.g., two Perfect Maps for *S* = {(A ⊥ B | C)}



I-Equivalence (1 of 2)

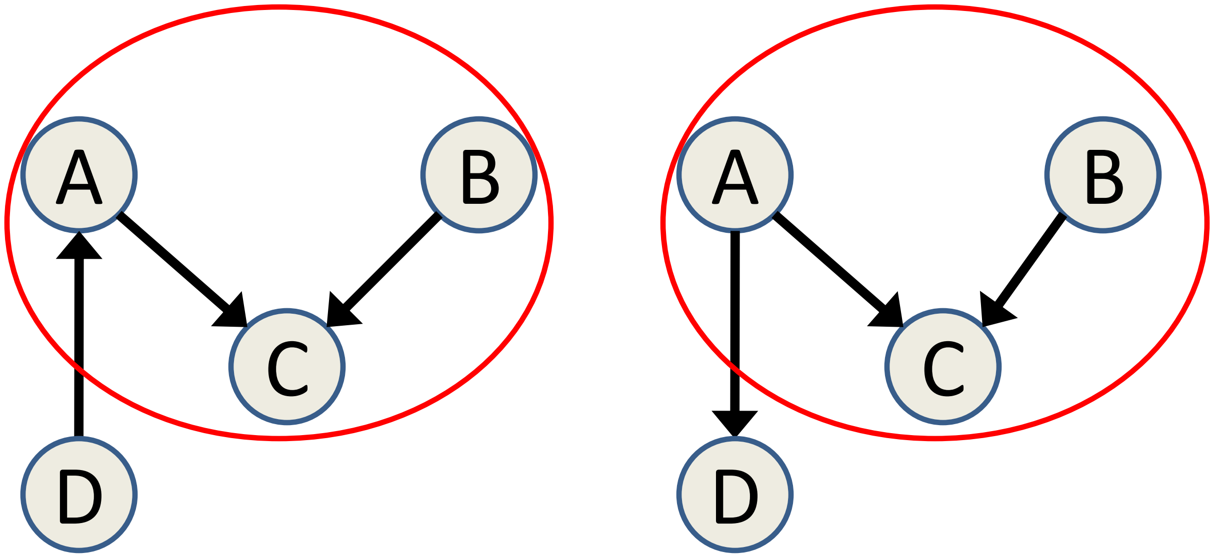
- Two graphs are ***I-Equivalent*** if they imply identical sets of independence assertions
-





I-Equivalence (2 of 2)

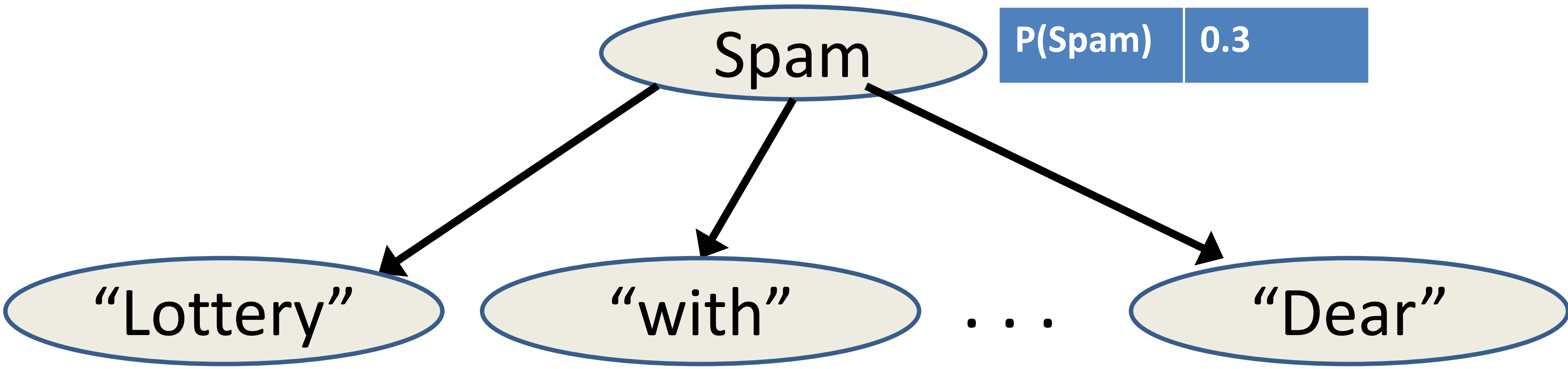
- Two graphs are I-Equivalent *iff* they have the same
  - *Skeleton:* graph ignoring edge direction
  - *Immoralities:* v-structures without direct edge between parents



Sidenote: Naïve Bayes Net

- NB assumes features conditionally indep. given the class:

P(Spam)	0.3



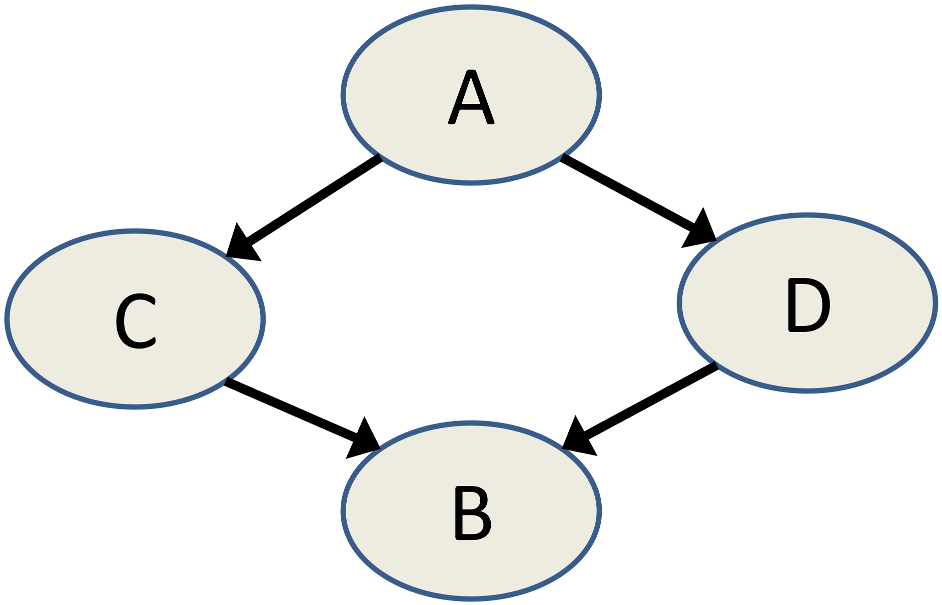
Spam	P(“Lottery” | Spam)
T	0.04
F	0.01

Spam	P(“with” | Spam)
T	0.6
F	0.59

Spam	P(“Lottery” | Spam)
T	0.24
F	0.30

Limitations of Bayesian Networks

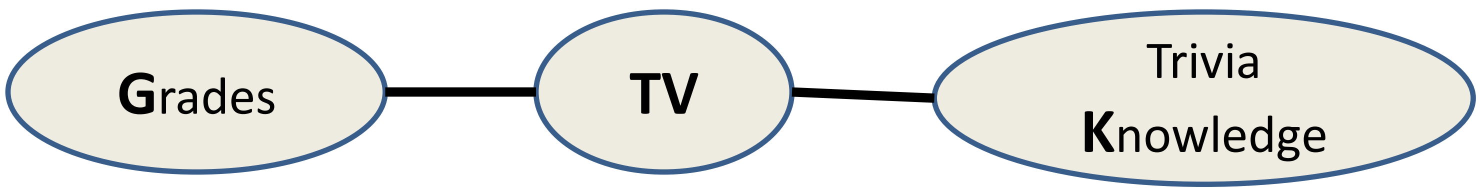
- Perfect Map for $\{(A \perp B \mid C, D), (C \perp D \mid A, B)\}$?



- Not possible! Bayes Nets can't express all possible sets of independence assertions.

Alternative: Markov Networks

- Undirected Graphical Model
  - No CPTs. Uses **potential functions** ϕ_c defined over cliques
- $P(\boldsymbol{x}) = \dfrac{\prod_c \phi_c(\boldsymbol{x}_c)}{Z}$ $\qquad Z = \sum_{\boldsymbol{x}} \prod_c \phi_c(\boldsymbol{x}_c)$

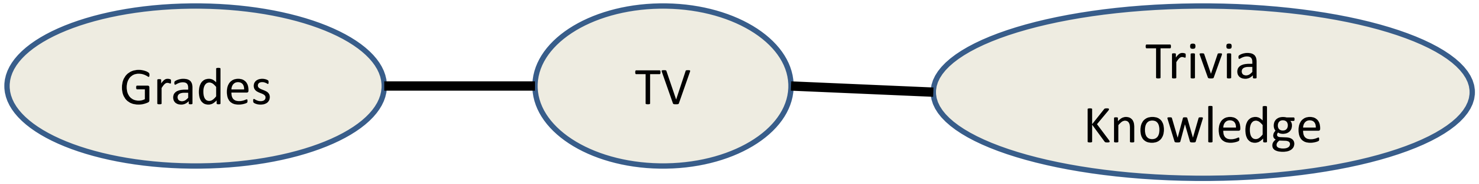


Grades	TV	ϕ_1(G, TV)
Bad	None	2.0
Good	None	3.0
Bad	Lots	3.0
Good	Lots	1.0

TV	Trivia Knowledge	ϕ_2(TV, K)
None	Weak	2.0
Lots	Weak	1.0
None	Strong	1.5
Lots	Strong	3.0

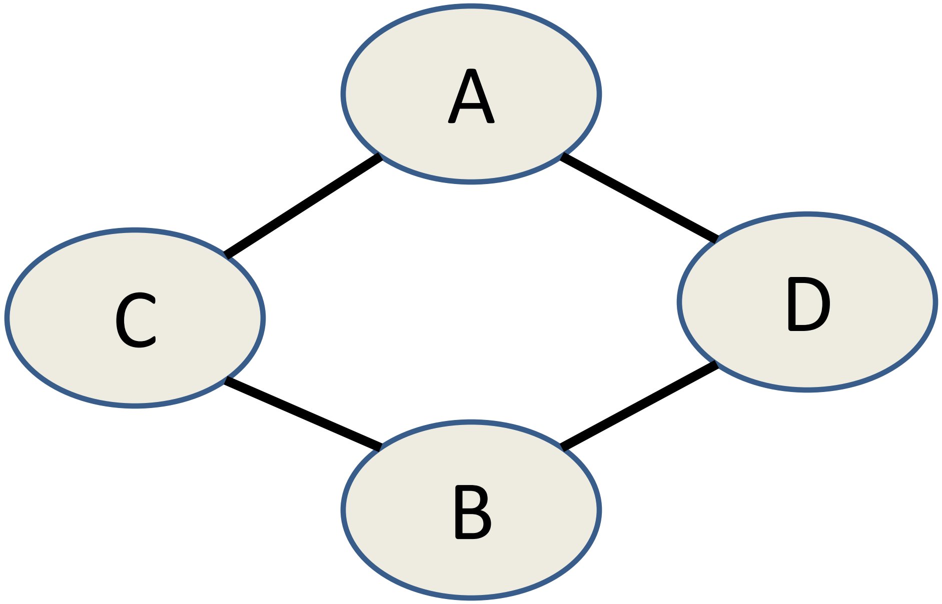
Markov Nets Independence Assertions

- Instead of D-separation, simply graph separation
  - So (Grades $\perp$ Trivia Knowledge | TV)



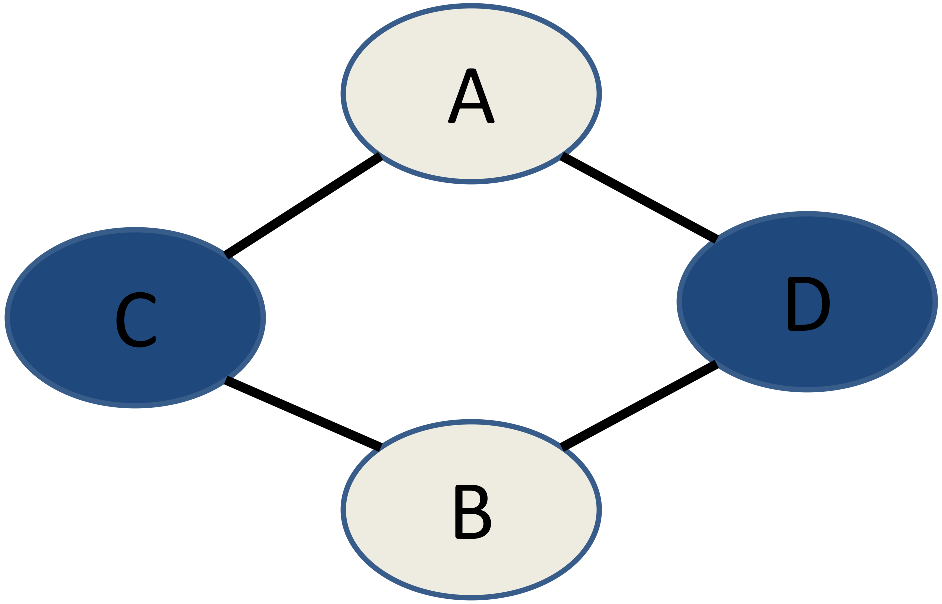
Expressivity of Markov Networks

- Perfect Map for $\{(A \perp B \mid C, D), (C \perp D \mid A, B)\}$?



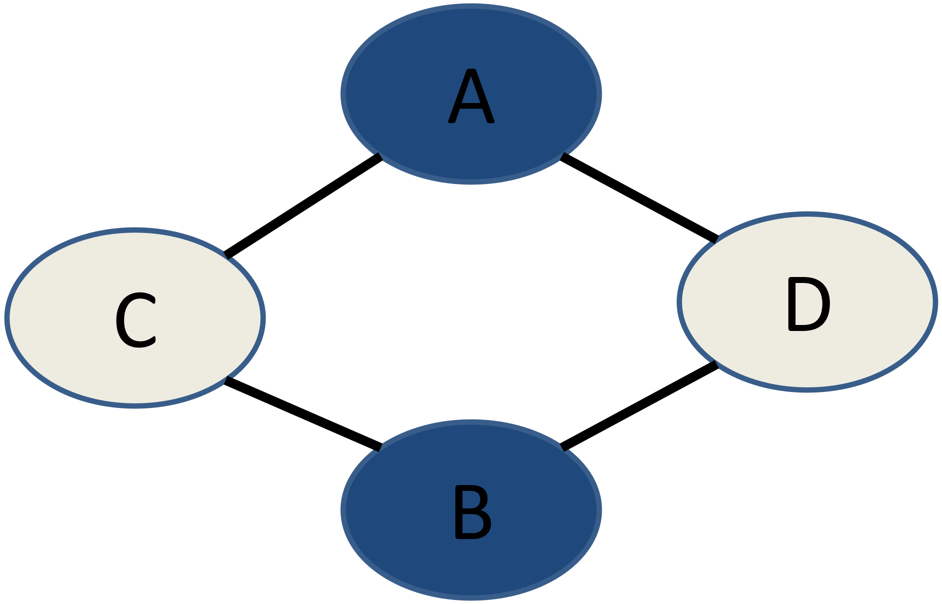
Expressivity of Markov Networks

- Perfect Map for {**(A ⊥ B | C, D)**, (C ⊥ D | A, B)}?



Expressivity of Markov Networks

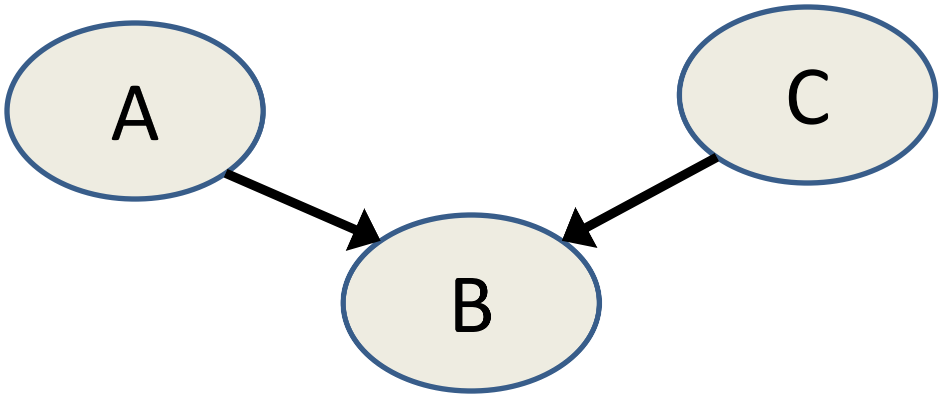
- Perfect Map for {(A ⊥ B | C, D), **(C ⊥ D | A, B)**}?



- Markov Nets *can* capture these independence assertions

But…

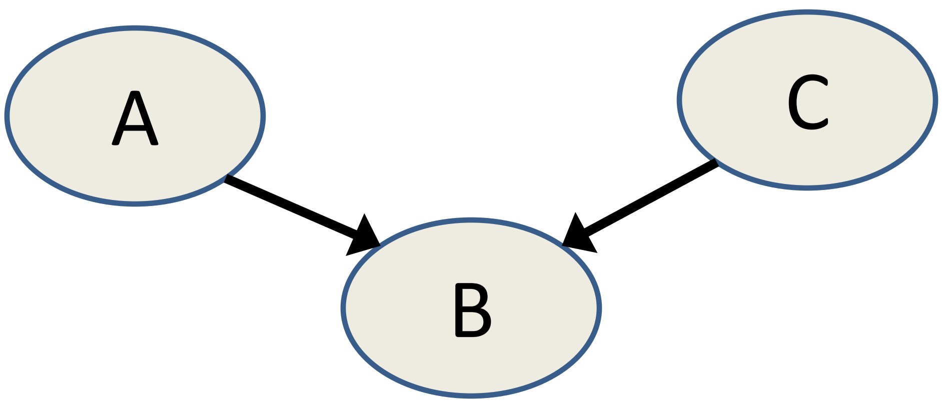
- How about $(A \perp C) \in S$, but $(A \perp C \mid B) \notin S$?



- Can’t be captured perfectly in Markov Networks
- If graph separation -> conditional independence, new knowledge can only **remove** dependencies

Bayesian Networks => Markov Networks

- Markov Nets can encode independences that Bayes Nets cannot, and vice-versa
- To convert from BN to MN, “moralize”:



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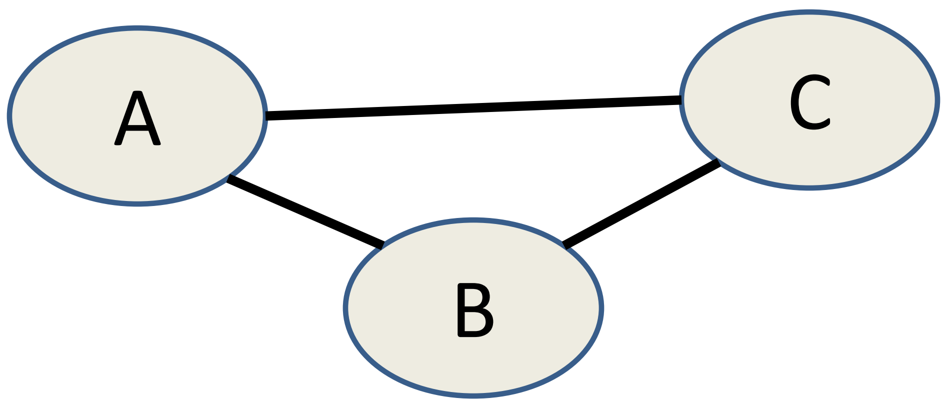

Markov Net Applications

- Best when no clear, directed causal structure
  - E.g. statistical physics, text, social networks, image analysis (e.g. segmentation, below)

Zoltan Kato http://www.inf.u-szeged.hu/ipcg/projects/RJMCMC.html