#### Markov Networks

Doug Downey Northwestern EECS 395/495 Fall 2011

#### First: Perfect Maps and I-Equivalence

- Perfect Map for S: A graph for a set S of independence assertions, i.e. statements of the form (X \(\begin{array}{c} Y \| Z) \)
- E.g., two Perfect Maps for  $S = \{(A \perp B \mid C)\}$



# I-Equivalence (1 of 2)

• Two graphs are *I-Equivalent* if they imply identical sets of independence assertions



# I-Equivalence (2 of 2)

- Two graphs are I-Equivalent *iff* they have the same
  - *Skeleton:* graph ignoring edge direction
  - Immoralities: v-structures without direct edge between parents



#### Sidenote: Naïve Bayes Net

• NB assumes features conditionally indep. given the class:



# Limitations of Bayesian Networks

• Perfect Map for {(A  $\perp$  B | C, D), (C  $\perp$  D | A, B)}?



• Not possible! Bayes Nets can't express all possible sets of independence assertions.

# Alternative: Markov Networks

• Undirected Graphical Model – No CPTs. Uses **potential functions**  $\phi_c$  defined over cliques





#### Markov Nets Independence Assertions

- Instead of D-separation, simply graph separation
  - So (Grades  $\perp$  Trivia Knowledge | TV)



## Expressivity of Markov Networks

• Perfect Map for {(A  $\perp$  B | C, D), (C  $\perp$  D | A, B)}?



## Expressivity of Markov Networks

• Perfect Map for { $(A \perp B \mid C, D)$ ,  $(C \perp D \mid A, B)$ }?



## Expressivity of Markov Networks

• Perfect Map for {(A ⊥ B | C, D), (C ⊥ D | A, B)}?



Markov Nets *can* capture these independence assertions

#### But...

• How about  $(A \perp C) \in S$ , but  $(A \perp C \mid B) \notin S$ ?



- Can't be captured perfectly in Markov Networks
- If graph separation -> conditional independence, new knowledge can only remove dependencies

## Bayesian Networks => Markov Networks

- Markov Nets can encode independences that Bayes Nets cannot, and vice-versa
- To convert from MN to BN, "moralize":



## Bayesian Networks => Markov Networks

- Markov Nets can encode independences that Bayes Nets cannot, and vice-versa
- To convert from MN to BN, "moralize":



## **Markov Net Applications**

- Best when no clear, directed causal structure
  - E.g. statistical physics, text, social networks, image analysis (e.g. segmentation, below)



Zoltan Kato http://www.inf.u-szeged.hu/ipcg/projects/RJMCMC.html