Machine Learning

Greedy Local Search

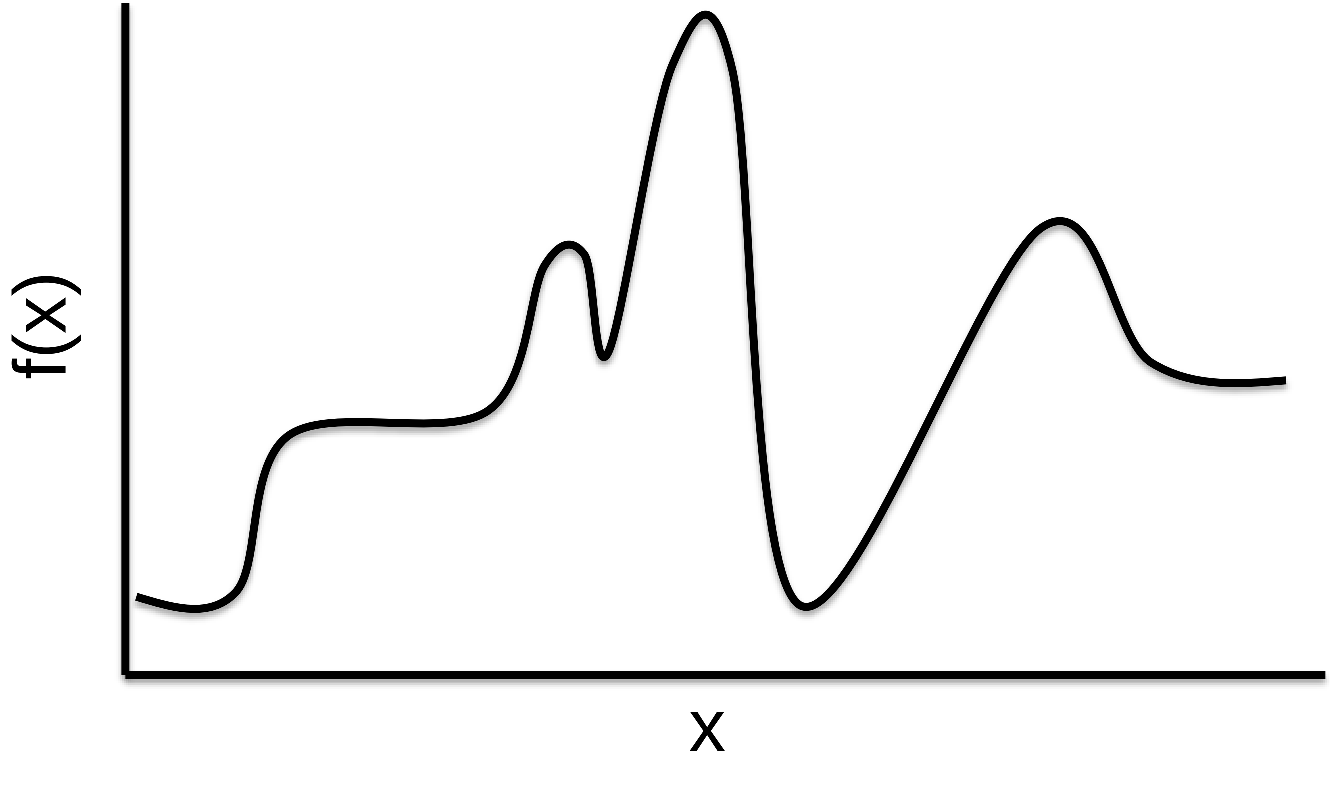
With slides from Bryan Pardo, Stuart Russell

ML in a Nutshell

- Every machine learning algorithm has three components:
  - Representation
    - E.g., Decision trees, instances
  - Evaluation
    - E.g., accuracy on test set
  - **Optimization**
    - How do you **find** the best hypothesis?

Hill-climbing (greedy local search)

$$\text{find } \; x_{\max} = \arg\max_{x \in X}(f(x))$$

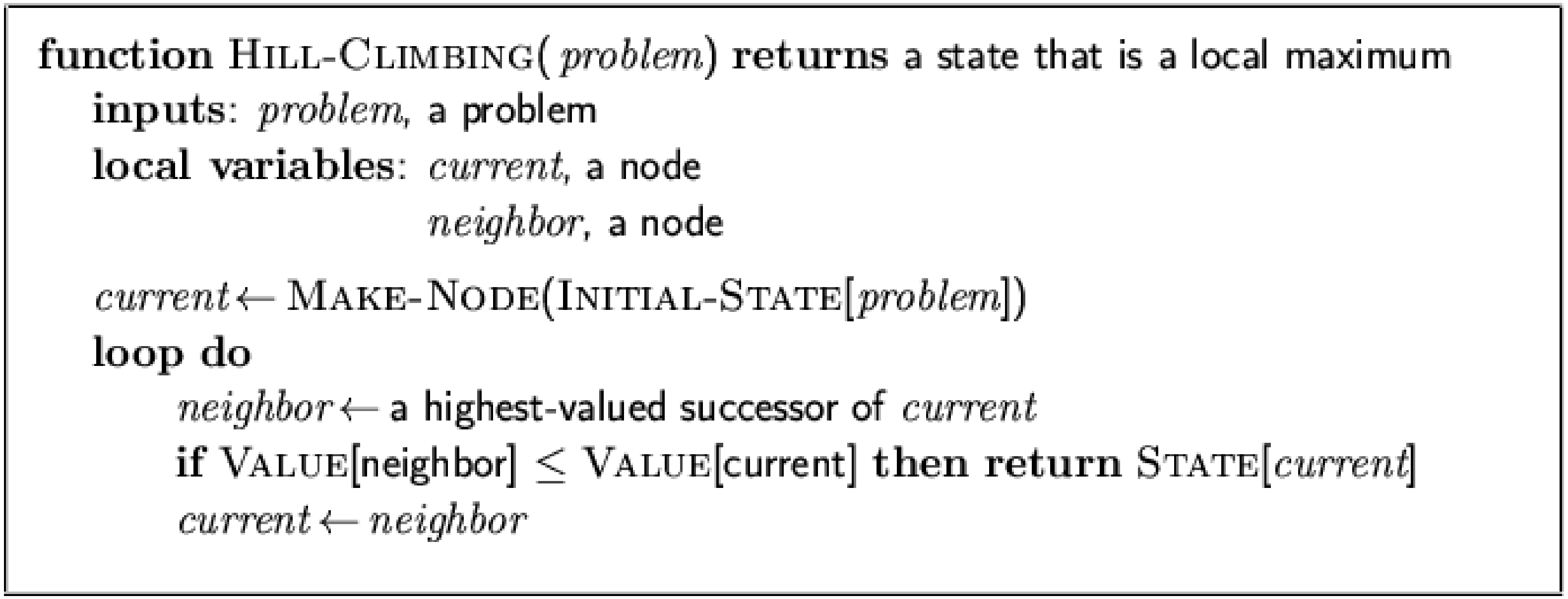


Greedy local search needs

- A “successor” function
  - Says what states I can reach from the current one.
  - Often implicitly a distance measure.
- An objective (error) function
  - Tells me how good a state is
- Enough memory to hold
  - The best state found so far
  - The current state
  - The state it’s considering moving to

Hill-climbing search

- "Like climbing Everest in thick fog with amnesia"

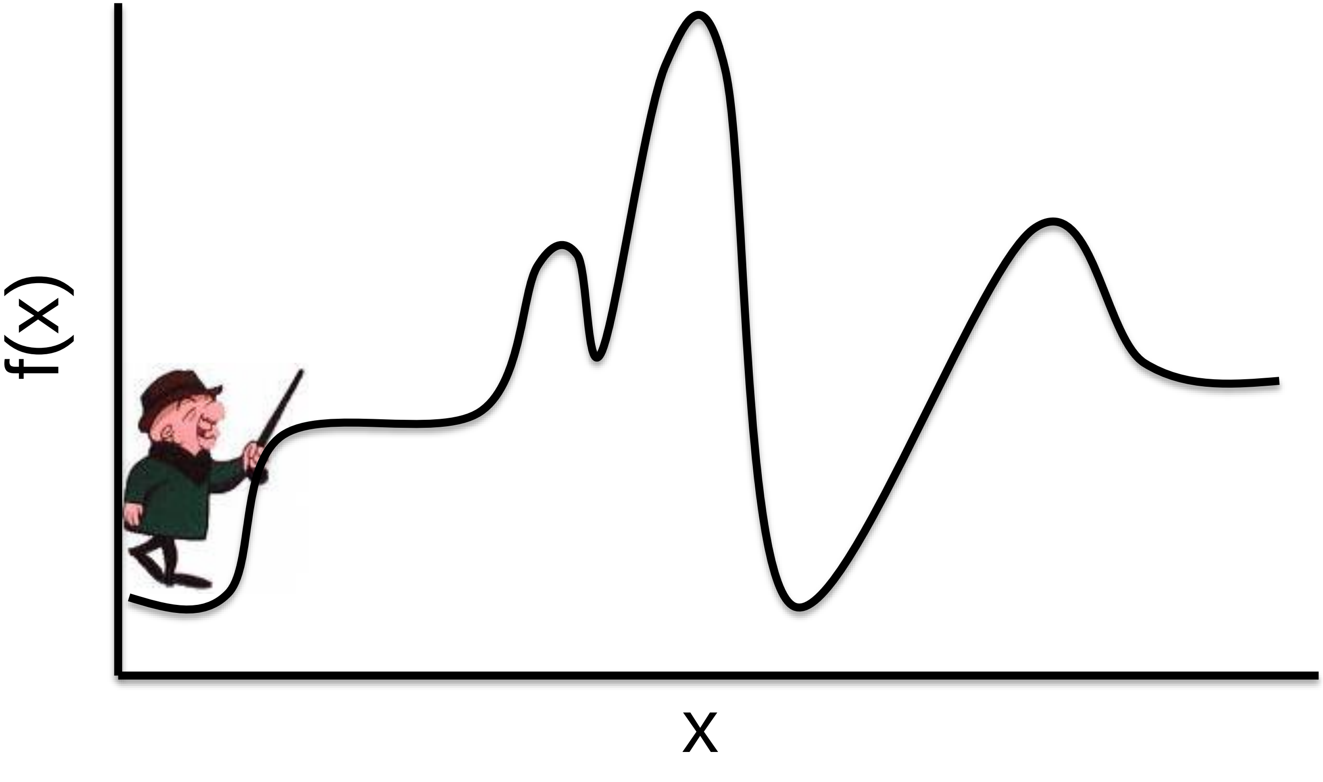


```
function HILL-CLIMBING(problem) returns a state that is a local maximum
    inputs: problem, a problem
    local variables: current, a node
                     neighbor, a node

    current $\leftarrow$ MAKE-NODE(INITIAL-STATE[problem])
    loop do
        neighbor $\leftarrow$ a highest-valued successor of current
        if VALUE[neighbor] $\leq$ VALUE[current] then return STATE[current]
        current $\leftarrow$ neighbor
```

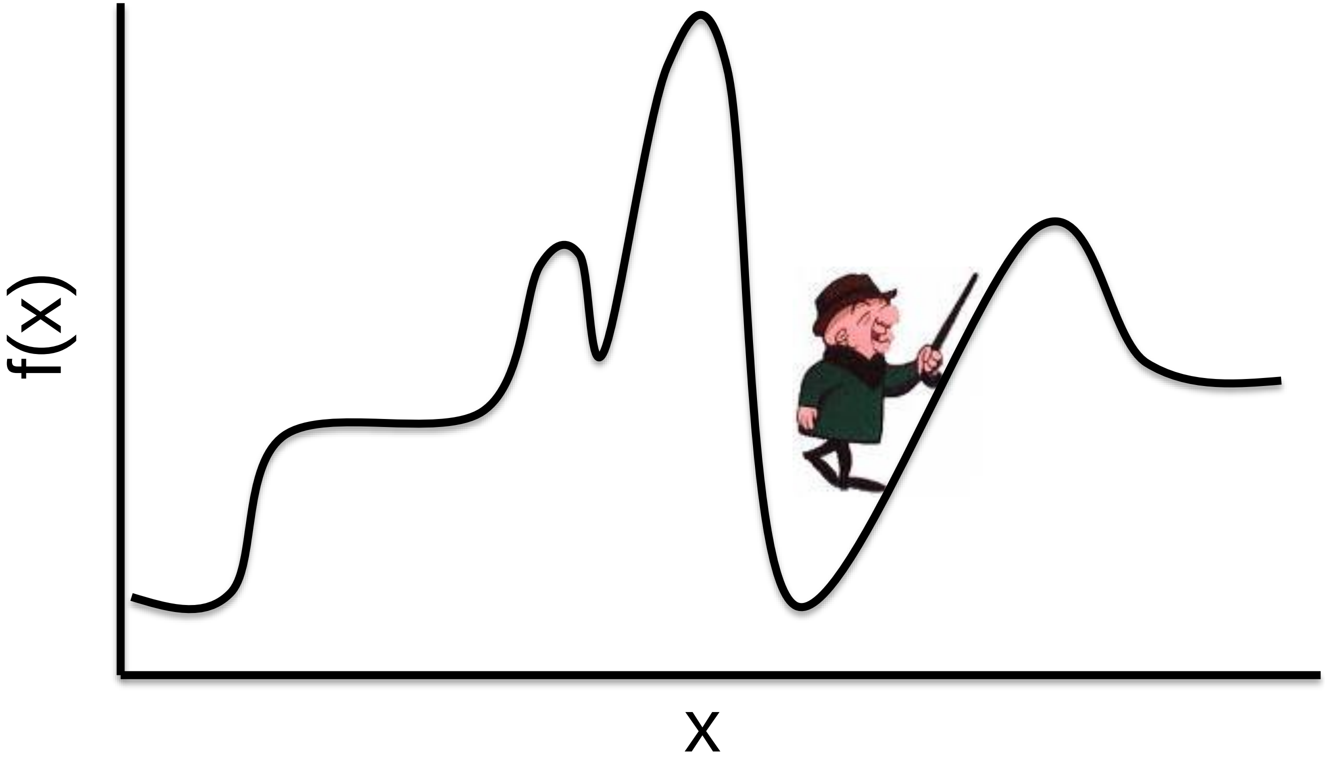
Hill-climbing (greedy local search)

"Like climbing Everest in thick fog with amnesia‟



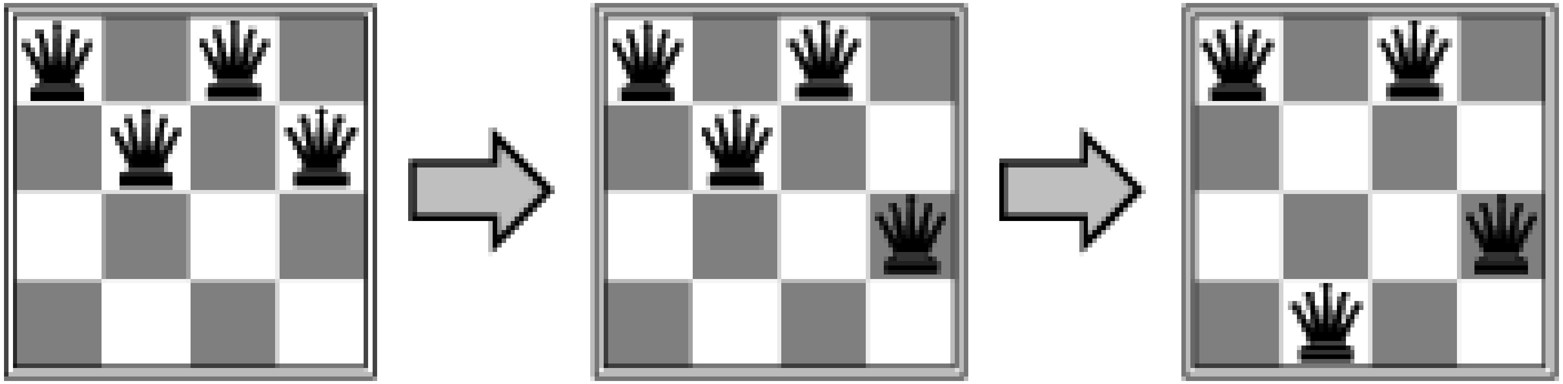
Hill-climbing (greedy local search)

It is easy to get stuck in local maxima



Example: *n*-queens

- Put *n* queens on an *n* × *n* board with no two queens on the same row, column, or diagonal
-



Greedy local search needs

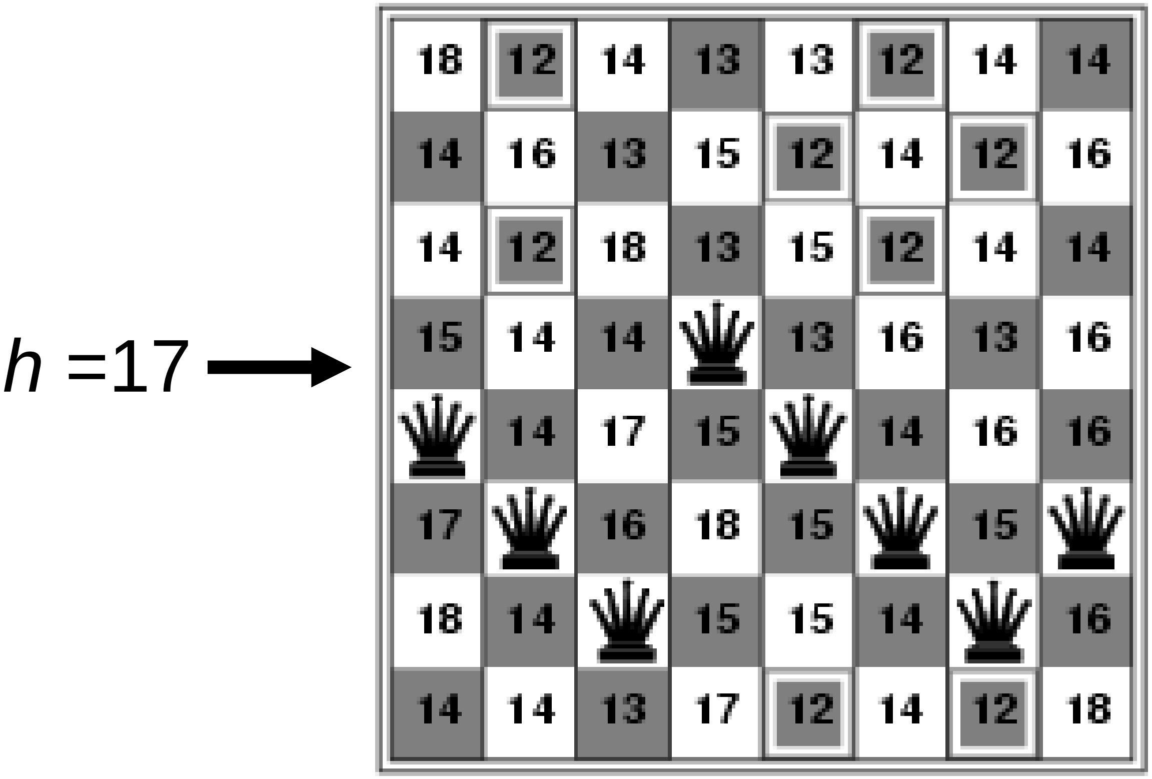
- A “successor” (distance?) function

  Any board position that is reachable by moving one queen in her column.

- An optimality (error?) measure

  How many queen pairs can attack each other?

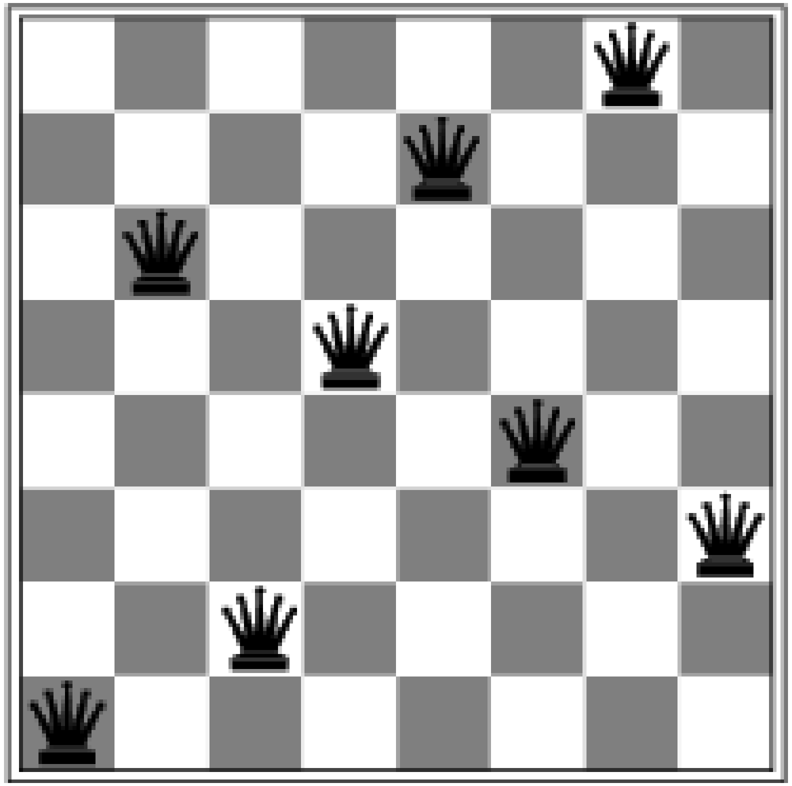
Hill-climbing search: 8-queens problem



18	12	14	13	13	12	14	14
14	16	13	15	12	14	12	16
14	12	18	13	15	12	14	14
15	14	14	♛	13	16	13	16
♛	14	17	15	♛	14	16	16
17	♛	16	18	15	♛	15	♛
18	14	♛	15	15	14	♛	16
14	14	13	17	12	14	12	18

- *h* = number of pairs of queens that are attacking each other, either directly or indirectly

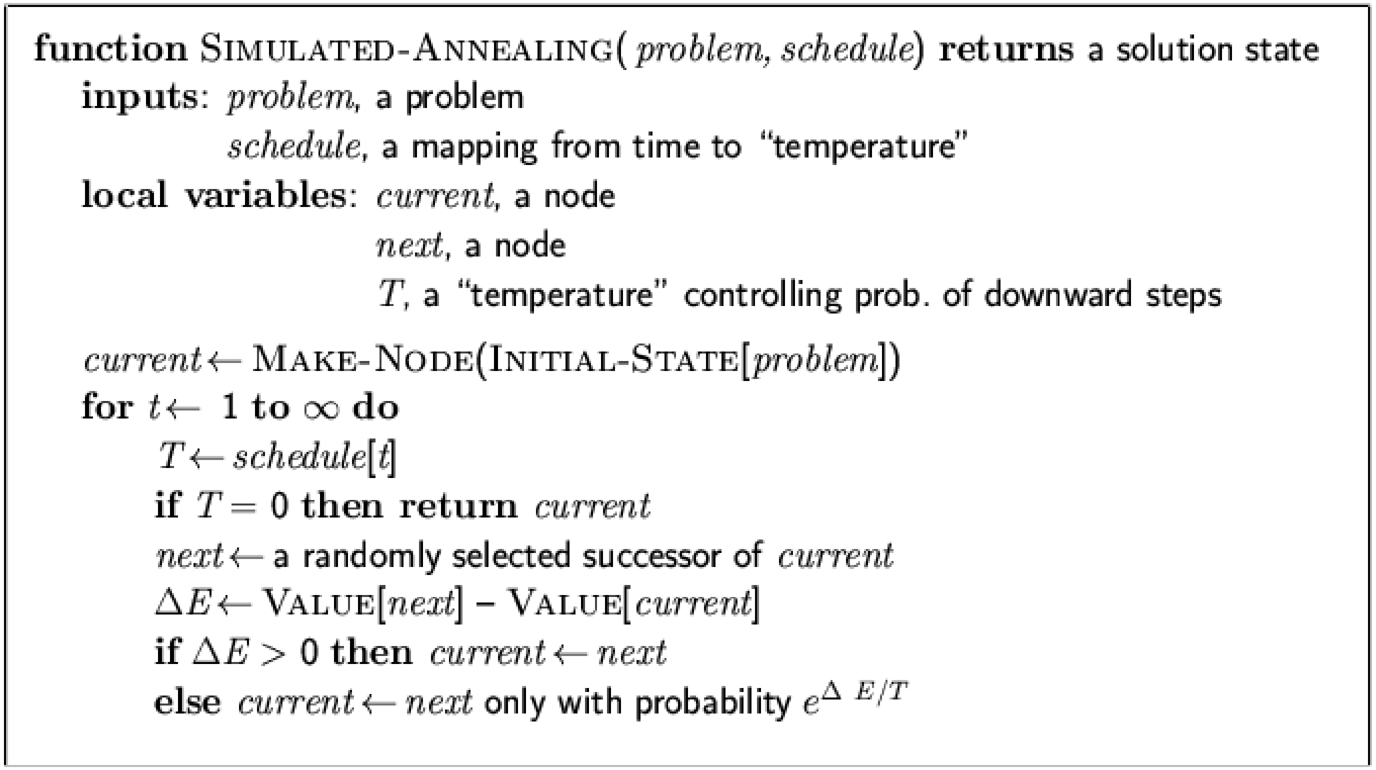
Hill-climbing search: 8-queens problem



- A local minimum with *h = 1*

Simulated annealing search

- Idea: escape local maxima by allowing some "bad" moves but gradually decrease their frequency



```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
    inputs: problem, a problem
            schedule, a mapping from time to "temperature"
    local variables: current, a node
                     next, a node
                     T, a "temperature" controlling prob. of downward steps

    current ← MAKE-NODE(INITIAL-STATE[problem])
    for t ← 1 to ∞ do
        T ← schedule[t]
        if T = 0 then return current
        next ← a randomly selected successor of current
        ΔE ← VALUE[next] – VALUE[current]
        if ΔE > 0 then current ← next
        else current ← next only with probability $e^{\Delta E/T}$
```

Properties of simulated annealing

- One can prove: If *T* decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1

- Widely used in VLSI layout, airline scheduling, etc

Let’s look at a demo

Results on 8-queens

	Random	Sim Anneal	Greedy
	600+	173	4
	15	119	4
	154	114	5
Average	**256+**	**135**	**4**

- Note: on other problems, your mileage may vary

Continuous Optimization

- Many AI problems require optimizing a function $f(\mathbf{x})$, which takes continuous values for input vector $\mathbf{x}$

- Huge research area

- Examples:
  - **Machine Learning**
  - Signal/Image Processing
  - Computational biology
  - Finance
  - Weather forecasting
  - Etc., etc.

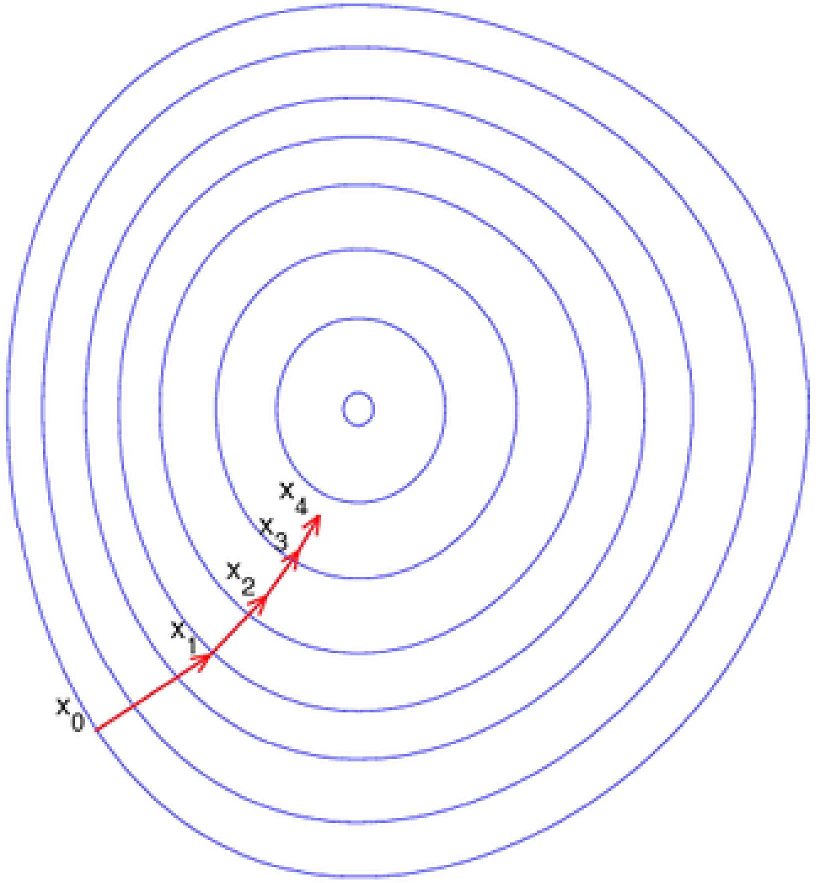
Local beam search

- Keep track of *k* states rather than just one
- Start with *k* randomly generated states
- At each iteration, all the successors of all *k* states are generated
- If any one is a goal state, stop; else select the *k* best successors from the complete list and repeat.

Gradient Ascent

- Idea: move in direction of steepest ascent (gradient)

- $\mathbf{x}_k = \mathbf{x}_{k\text{-}1} + \eta \nabla f(\mathbf{x}_{k\text{-}1})$



x	f(x)	gradient	xnew
5.000	9.000	6.000	4.000
4.000	4.000	4.000	3.333
3.333	1.778	2.667	2.889
2.889	0.790	1.778	2.593
2.593	0.351	1.185	2.395
2.395	0.156	0.790	2.263
2.263	0.069	0.527	2.176
2.176	0.031	0.351	2.117
2.117	0.014	0.234	2.078
2.078	0.006	0.156	2.052
2.052	0.003	0.104	2.035
2.035	0.001	0.069	2.023
2.023	0.001	0.046	2.015
2.015	0.000	0.031	2.010

Types of Optimization

- Linear vs. non-linear
- Analytic vs. Empirical Gradient
- Convex vs. non-convex
- Constrained vs. unconstrained

Continuous Optimization in Practice

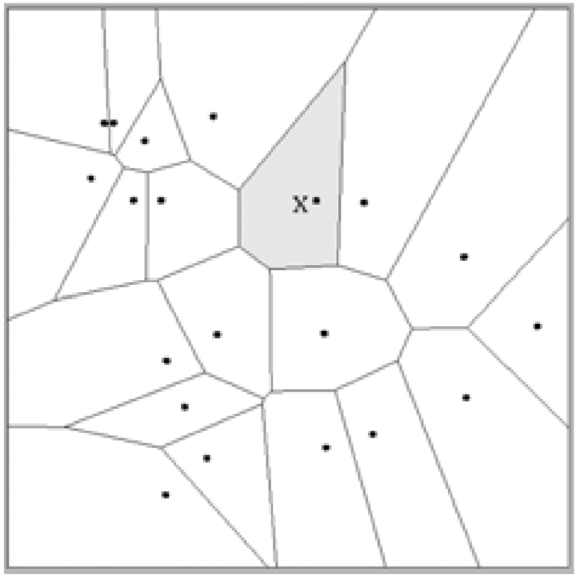
- *Lots* of previous work on this
- Use packages

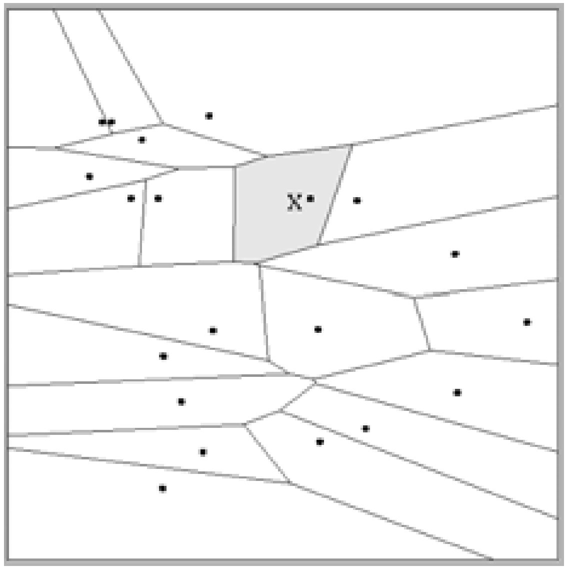
Rules of Thumb: Gradient Descent

- Stochastic vs. Batch
  - Try stochastic first

- Analytic Gradients are hard to debug
  - Use packages with gradients built in (e.g. Tensorflow)
  - Do gradient checking

Final example: weights in NN





$$d(x,y) = (x_1 - y_1)^2 + (x_2 - y_2)^2$$

$$d(x,y) = (x_1 - y_1)^2 + (3x_2 - 3y_2)^2$$

Reading

- Gradient Descent
  - See e.g. the python example midway down the page
- Previous:
  - Nearest neighbor (Elements of Statistical Learning 2.3, 13.3)