

DUE Friday, April 27 at 11:59PM. Late assignments will not be graded.

1. Prove or find a counterexample for all statements below (1/2pt each)
 - a. $P \wedge \neg P$ is valid
 - b. $(A \rightarrow B) \vDash B$
 - c. if $(A \vDash B)$ and $(B \vDash C)$ then $((A \text{ or } B) \vDash C)$
 - d. $A \vee \neg(B \wedge \neg C) \vDash (A \vee C)$
 - e. $\neg(A \wedge B) \wedge (A \rightarrow B)$ is satisfiable
 - f. There are the same number of models for the statement $(A \wedge (B \vee C))$ as there are for the statement $((A \rightarrow \neg B) \wedge A)$
2. (3x1/2 points) Translate the following first order logic sentences to English. Predicates and functions carry the obvious meanings.
 - a. mascot(Northwestern, Willie Wildcat)
 - b. $\exists x \exists y \neg \text{int}(\text{div}(x, y))$
 - c. $\forall x \forall y \forall z (\text{odd}(\text{sum}(x, y)) \wedge \text{odd}(z)) \rightarrow \neg \text{odd}(\text{sum}(\text{sum}(x, y), z))$
3. (3x1/2 points) Translate the following English sentences to first order logic. Name predicates and functions such that their meanings are obvious.
 - a. the meaning of life, the universe, and every-thing is 42
 - b. Ron, Champ, Brick, and Brian are on the Channel 6 News Team
 - c. For any two integers x and y , the product of x and y is an integer
4. 6 pts
 - a. If a threatening envoy is kicked down a well, then "This is Sparta!"
 - b. If an envoy is carrying the skulls of kings, then he is threatening
 - c. Xerxes is the King of Persia
 - d. Every envoy from Xerxes will carry the skulls of kings
 - e. Leonidas is the King of Sparta
 - f. If Leonidas is sent a threatening envoy, he will kick the envoy down a well
 - g. Xerxes has sent an envoy to Leonidas
 - h. If B is the enemy of A and B is also the enemy of C, then A and C are friends
 - i. If the king of A sends a threatening envoy to the king of B, A is the enemy of B
 - j. Xerxes has sent an envoy to the King of Arcadia

Translate each of the above and below English sentences to first-order logic (1/5 of a point per translation). Using the above knowledge base only, i.e. not assuming any other real world facts, prove (2 points per proof) by resolution that

- k. "This is Sparta!"
- l. Arcadia and Sparta are friends